8.1
Notes

TRIG FUNCTIONS OF REAL NUMBERS

OBJECTIVES: 1) Use the remaining Pythagorean identities to simplify trig expressions.
2) Use opposite angle identities to calculate values of trig functions.

TRIG FUNCTIONS OF REAL NUMBERS: DEFINITIONS

$\cos t = x$ $\quad$ $\sin t = y$ $\quad$ $\tan t = \dfrac{y}{x}$

$\sec t = \dfrac{1}{x}$ $\quad$ $\csc t = \dfrac{1}{y}$ $\quad$ $\cot t = \dfrac{x}{y}$

PYTHAGOREAN IDENTITIES:

1) $\sin^2\theta + \cos^2\theta = 1$ $\quad$ $\sin^2 t + \cos^2 t = 1$

2) $1 + \cot^2 t = \csc^2 t$ 3) $\tan^2 t + 1 = \sec^2 t$

1) Simplify: $\dfrac{u}{\sqrt{u^2-1}}$ by substituting $u = \sec\theta$

(Assume $0 < \theta < \dfrac{\pi}{2}$.)

$= \dfrac{\sec\theta}{\sqrt{\sec^2\theta - 1}}$

$= \dfrac{\sec\theta}{\sqrt{\tan^2\theta}}$

$= \dfrac{\sec\theta}{|\tan\theta|}$

$= \dfrac{\sec\theta}{\tan\theta}$

$= \dfrac{1}{\cos\theta} \cdot \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{1}{\sin\theta}$

$= \boxed{\csc\theta}$

2) Simplify: $\dfrac{\sec t + \sec t \tan^2 t}{\cot^2 t - \csc^2 t}$

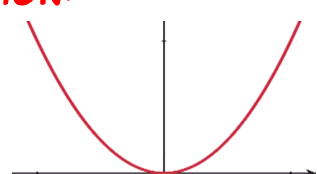
$\dfrac{\sec t(1 + \tan^2 t)}{\cot^2 t - \csc^2 t}$ $\Rightarrow \tan^2 t + 1 = \sec^2 t$ $\Rightarrow \csc^2 t - \cot^2 t = 1$

$= \dfrac{\sec t(\sec^2 t)}{-1}$

$= \boxed{-\sec^3 t}$

EVEN FUNCTION:

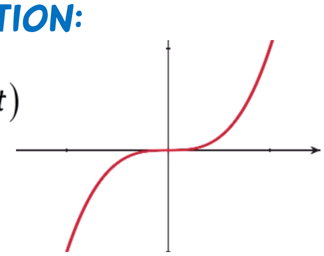
$f(-t) = f(t)$
for all t in the domain of f



Symmetric about the y-axis

ODD FUNCTION:

$f(-t) = -f(t)$
for all t in the domain of f



Symmetric about the origin.

OPPOSITE ANGLE IDENTIES (PROVE IT: PART III)

$\cos(-t) = \cos(t)$ (cosine is an even function)

$\sin(-t) = -\sin(t)$ (sine is an odd function)

$\tan(-t) = -\tan(t)$ (tangent is an odd function)

Using the opposite angle identities:

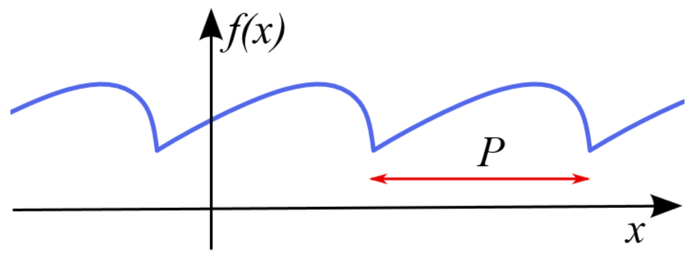
3) If $\cos m = -.45$ find $\cos(-m)$. $\cos(-m) = -.45$

4) If $\tan p = 1.2$ find $\tan(-p)$. $\tan(-p) = -1.2$

5) If $\cos g = .36$ find $\cos^2(-g) + \sin^2(-g)$. $\cos^2(-g) + \sin^2(-g) = 1$

PERIODIC FUNCTION:

A function that a **periodic function** is a function repeats its values in regular intervals or periods.



PERIODICITY:

Every 2π represents a complete trip around the unit circle.

$$\cos(t + 2\pi k) = \cos(t) \qquad \sin(t + 2\pi k) = \sin(t)$$

6) $\cos(-19\pi)$

$$\cos(-19\pi) = \cos(-\pi - 2\pi \cdot 9)$$
$$= \cos(-\pi) = \cos(\pi)$$
$$= \boxed{-1}$$

7) $\sin\left(\frac{5\pi}{2}\right)$

$$\sin\left(\frac{5\pi}{2}\right) = \sin\left(\frac{1}{2}\pi + 2\pi\right)$$
$$= \sin\left(\frac{1}{2}\pi\right) = \boxed{1}$$