(PART 1) THE ADDITION FORMULAS

OBJECTIVES: 1) Use the cosine and sine addition formulas to simplify expressions.

The Distributive Property does not hold true for functions: $a(b+c)=ab+ac$ but $f(a+b)\neq f(a)+f(b)$

THE ADDITION FORMULAS FOR SINE AND COSINE

$\sin(s+t)=\sin s\cos t+\cos s\sin t$

$\sin(s-t)=\sin s\cos t-\cos s\sin t$

$\cos(s+t)=\cos s\cos t-\sin s\sin t$

$\cos(s-t)=\cos s\cos t+\sin s\sin t$

* The above formulas are proven in Part 4 and Part 5 of your Prove It Notes.

1) Simplify the expression $\cos 2\theta\cos\theta+\sin 2\theta\sin\theta$.

$\cos(2\theta-\theta)=\boxed{\cos\theta}$

2) Simplify $\cos(\theta-\pi)$

$\cos(\theta-\pi)=\cos\theta\cos\pi+\sin\theta\sin\pi$

$=\cos\theta(-1)+\sin\theta\cdot 0$

$=\boxed{-\cos\theta}$

3) Simplify the expression $\sin\theta\cos 2\theta-\cos\theta\sin 2\theta$.

$\sin(\theta-2\theta)=\sin(-\theta)=\boxed{-\sin\theta}$

4) Simplify $\sin\left(x-\frac{\pi}{2}\right)$

$\sin\left(x-\frac{\pi}{2}\right)=\sin x\cdot\cos\left(\frac{\pi}{2}\right)-\cos x\cdot\sin\frac{\pi}{2}$

$=\sin x\cdot 0-\cos x\cdot 1$

$\boxed{=-\cos x}$

5) Find the exact value of $\cos 105°$.

$\cos(45+60)=\cos 45\cos 60-\sin 45\sin 60$

$=\frac{\sqrt{2}}{2}\cdot\frac{1}{2}-\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}$

$=\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{4}$

$=\boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

6) Find the exact value of $\cos 15°$

$\cos 15=\cos(45-30)$

$=\cos 45\cos 30+\sin 45\sin 30$

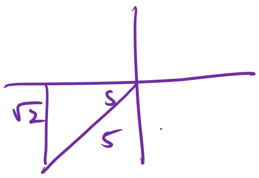
$=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\cdot\frac{1}{2}$

$=\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}$

$\boxed{=\frac{\sqrt{6}+\sqrt{2}}{4}}$

7) If $\sin s = -\dfrac{\sqrt{2}}{5}$, where $\left(\pi < s < \dfrac{3\pi}{2}\right)$, and $\sin t = \dfrac{2}{3}$, where $\left(0 < t < \dfrac{\pi}{2}\right)$, find $\sin(s+t)$.

$\sin(s+t) = \sin s \cos t + \cos(s)\sin t$ Find $\cos(s)$ and $\cos(t)$!



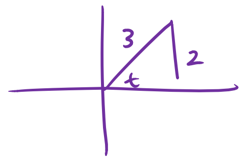
$\sqrt{2}^2 + x^2 = 5^2$

$2 + x^2 = 25$

$x^2 = 23$

$x = \pm\sqrt{23}$

$\cos s = -\dfrac{\sqrt{23}}{5}$



$x^2 + 2^2 = 3^2$

$x^2 = 5$

$x = \pm\sqrt{5}$

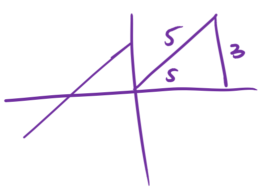
$\cos t = \dfrac{\sqrt{5}}{3}$

$\sin(s+t) = -\dfrac{\sqrt{2}}{5} \cdot \dfrac{\sqrt{5}}{3} + \dfrac{2}{3} \cdot \dfrac{-\sqrt{23}}{5}$

$-\dfrac{\sqrt{10}}{15} - \dfrac{2\sqrt{23}}{15} = \boxed{\dfrac{-\sqrt{10} - 2\sqrt{23}}{15}}$

8) If $\sin s = \dfrac{3}{5}$, where $\left(0 < s < \dfrac{\pi}{2}\right)$, and $\sin t = -\dfrac{\sqrt{3}}{4}$, where $\left(\pi < t < \dfrac{3\pi}{2}\right)$ find $\sin(s-t)$.

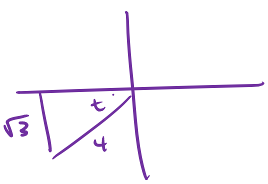
$\sin(s-t) = \sin s \cos t - \cos s \sin t$



$x^2 + 3^2 = 5^2$

$x = 4$

$\cos(s) = \dfrac{4}{5}$



$\sqrt{3}^2 + x^2 = 4^2$

$3 + x^2 = 16$

$x^2 = 13$

$x = -\sqrt{13}$

$\cos t = -\dfrac{\sqrt{13}}{4}$

$\sin(s-t) = \dfrac{3}{5} \cdot -\dfrac{\sqrt{13}}{4} - \dfrac{4}{5} \cdot -\dfrac{\sqrt{3}}{4}$

$= -\dfrac{3\sqrt{13}}{20} + \dfrac{4\sqrt{3}}{20}$

$= \boxed{\dfrac{-3\sqrt{13} + 4\sqrt{3}}{20}}$