# (PART 2) THE ADDITION FORMULAS

**OBJECTIVES**: 1) Use the tangent addition formula to simplify expressions.
2) Verify trig identities using new strategies involving the addition formulas.

## PROVE TAN(S+T) AND TAN(S−T)

Begin with what we have already established in our previous section…

$$\tan(s+t) = \frac{\sin(s+t)}{\cos(s+t)} = \frac{\sin s\cos t + \cos s \sin t}{\cos s \cos t - \sin s \sin t}\left(\frac{\frac{1}{\cos s \cos t}}{\frac{1}{\cos s\cos t}}\right)$$

$$= \frac{\tan s + \tan t}{1-\tan s \tan t}$$

$$\boxed{\tan(s+t) = \frac{\tan s + \tan t}{1 - \tan s \cdot \tan t}}$$

$$\tan(s-t) = \tan(s+-t) = \frac{\tan s + \tan(-t)}{1-\tan s\cdot\tan(-t)}$$

$$= \frac{\tan s - \tan t}{1+\tan s \tan t}$$

$$\boxed{\tan(s-t) = \frac{\tan s - \tan t}{1+\tan s \tan t}}$$

### ADDITION FORMULAS FOR TANGENT:

$$\tan(s+t) = \frac{\tan s + \tan t}{1-\tan s \tan t} \qquad \tan(s-t) = \frac{\tan s - \tan t}{1+\tan s \tan t}$$

1) Simplify the expression $\dfrac{\tan\frac{\pi}{9} + \tan\frac{2\pi}{9}}{1-\tan\frac{\pi}{9}\tan\frac{2\pi}{9}}$

$$\tan\left(\frac{\pi}{9} + \frac{2\pi}{9}\right) = \tan\left(\frac{3\pi}{9}\right) = \tan\left(\frac{\pi}{3}\right)$$

$$= \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \boxed{\sqrt{3}}$$

2) $\cos(\alpha+\beta)\cos\beta + \sin(\alpha+\beta)\sin\beta$

$$\cos(\alpha+\beta+\beta) = \cos(\alpha+2\beta)$$

3) Find $\tan\left(\frac{\pi}{12}\right)$.

$$\tan\left(\frac{\pi}{12}\right) = \tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$\tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \frac{\tan\left(\frac{\pi}{3}\right) - \tan\left(\frac{\pi}{4}\right)}{1 + \tan\left(\frac{\pi}{3}\right)\left(\tan\frac{\pi}{4}\right)}$$

$$= \frac{\sqrt{3}-1}{1+\sqrt{3}} = \frac{(\sqrt{3}-1)^2}{1-3} = \frac{3-2\sqrt{3}+1}{-2} = \frac{4-2\sqrt{3}}{-2}$$

$$= \boxed{-2+\sqrt{3}}$$

4) Find $\tan\left(\frac{7\pi}{12}\right)$

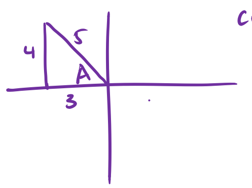
$$\tan\left(\frac{7\pi}{12}\right) = \tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$$

$$= \frac{\tan\left(\frac{\pi}{3}\right) + \tan\left(\frac{\pi}{4}\right)}{1 - \tan\left(\frac{\pi}{3}\right)\left(\frac{\pi}{4}\right)}$$

$$= \frac{\sqrt{3}+1}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}} = \frac{4+2\sqrt{3}}{-2} = \boxed{-2-\sqrt{3}}$$

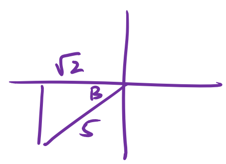
5) Given: $\sin(A) = \frac{4}{5}$ where $\frac{\pi}{2} < A < \pi$ and $\cos(B) = -\frac{\sqrt{2}}{5}$ where $\pi < B < \frac{3\pi}{2}$

Find $\tan(A+B)$.




$$\cos A = -\frac{3}{5}$$

$$\tan A = \frac{4}{-3}$$




$$x^2 + (\sqrt{2})^2 = 5^2$$

$$x^2 + 2 = 25$$

$$x^2 = 23$$

$$x = \pm\sqrt{23}$$

$$\sin B = \frac{-\sqrt{23}}{5}$$

$$\tan B = \frac{\sqrt{23}}{\sqrt{2}} = \frac{\sqrt{46}}{2}$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{-\frac{4}{3} + \frac{\sqrt{46}}{2}}{1 - \left(-\frac{4}{3}\right)\left(\frac{\sqrt{46}}{2}\right)}$$

$$= \frac{\frac{-8+3\sqrt{46}}{6}}{\frac{6+4\sqrt{46}}{6}} = \frac{-8+3\sqrt{46}}{6+4\sqrt{46}}$$

I'm stopping here. You can too. Sorry these numbers are nasty!

Verify the trig identity:

6) $\dfrac{\cos(s-t)}{\cos s \sin t} = \cot t + \tan s$

LHS:

$$\frac{\cos(s-t)}{\cos s \sin t} =$$

$$\frac{\cos s \cos t + \sin s \sin t}{\cos s \sin t} =$$

$$\frac{\cos s \cos t}{\cos s \sin t} + \frac{\sin s \sin t}{\cos s \sin t} =$$

$$\cot t + \tan s \quad \checkmark$$

7) $\sin(A-B) + \sin(A+B) = 2\sin A\cos B$

LHS.

$$\sin A \cos B - \cancel{\cos A \sin B} + \sin A \cos B + \cancel{\cos A \sin B} =$$

$$2\sin A \cos B = \quad \checkmark$$