(PART 1) THE DOUBLE-ANGLE FORMULAS

OBJECTIVES: 1) Use the double angle formulas to simplify expressions.

THE DOUBLE-ANGLE FORMULAS

$\sin 2\theta = 2\sin\theta\cos\theta$ $\qquad$ $\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\qquad$ $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

EQUIVALENT FORMS

$\cos 2\theta = 2\cos^2\theta - 1$ $\qquad$ $\cos 2\theta = 1 - 2\sin^2\theta$ $\qquad$ $\cos 2\theta = \dfrac{1+\cos 2\theta}{2}$ $\qquad$ $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

* The above formulas are proven in Part 6 of your Prove It Notes.

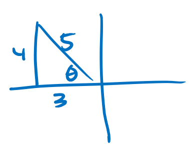
USING DOUBLE-ANGLE FORMULAS

1) If $\sin\theta = \dfrac{4}{5}$ and $\dfrac{\pi}{2} < \theta < \pi$, find the quantities $\sin 2\theta$ and $\cos 2\theta$.

$\cos\theta = -\dfrac{3}{5}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right)$



$= \boxed{-\dfrac{24}{25}}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2$

$= \boxed{-\dfrac{7}{25}}$

OR: $\cos 2\theta = 1 - 2\sin^2\theta$

$= 1 - 2\left(\frac{4}{5}\right)^2 = 1 - \frac{32}{25} = -\frac{7}{25}$

2) If $x = 4\sin\theta,\ 0 < \theta < \dfrac{\pi}{2}$, express $\sin 2\theta$ in terms of x.

$\frac{x}{4} = \sin\theta$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos\theta = \sqrt{1-\sin^2\theta}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\sin 2\theta = \cancel{2}\left(\frac{x}{4}\right)\sqrt{1-\sin^2\theta}$ $\leftarrow$ + since $0 < \theta < \frac{\pi}{2}$

$= 2\left(\frac{x}{4}\right)\sqrt{1 - \frac{x^2}{16}}$

$= \frac{x}{2}\sqrt{\frac{16-x^2}{16}} = \frac{x}{2}\cdot\frac{\sqrt{16-x^2}}{4} = \boxed{\dfrac{x\sqrt{16-x^2}}{8}}$

3) If $\cos\theta = \frac{3}{5}$ and $\frac{3\pi}{2} < \theta < 2\pi$, find the quantities $\sin\theta, \sin 2\theta$, and $\cos 2\theta$.

$\sin\theta = \boxed{\frac{-4}{5}}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(-\frac{4}{5}\right)\left(\frac{3}{5}\right)$

$\sin 2\theta = \boxed{-\frac{24}{25}}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \frac{9}{25} - \frac{16}{25}$

$\cos 2\theta = \boxed{\frac{-7}{25}}$

4) If $x = 3\sin\theta$, $\frac{\pi}{2} < \theta < \pi$, express:

a) $\cos 2\theta$ in terms of x

$\frac{x}{3} = \sin\theta$

$\cos 2\theta = \sin^2\theta - \cos^2\theta$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\cos 2\theta = 1 - 2\left(\frac{x}{3}\right)^2$

$\cos 2\theta = \boxed{1 - \frac{2x^2}{9}}$

b) $\sin 2\theta$ in terms of x

$\sin 2\theta = 2\sin\theta\cos\theta \leftarrow$ $\cos\theta$ is neg!

$= -2\left(\frac{x}{3}\right)\sqrt{1 - \frac{2x^2}{9}}$

$= -2\left(\frac{x}{3}\right)\frac{\sqrt{9-2x^2}}{3}$

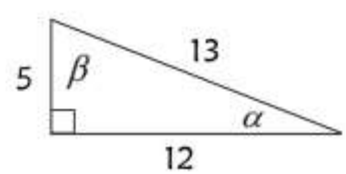
$= \boxed{\frac{-2x\sqrt{9-2x^2}}{9}}$

THE HALF–ANGLE FORMULAS

*Part 7 of your Prove It Notes

$$\sin\frac{s}{2} = \pm\sqrt{\frac{1-\cos s}{2}} \qquad \cos\frac{s}{2} = \pm\sqrt{\frac{1+\cos s}{2}} \qquad \tan\frac{s}{2} = \frac{\sin s}{1+\cos s}$$

Use an appropriate half-angle formula to evaluate each quantity.



5) $\sin\left(\frac{\beta}{2}\right)$

$\sin\left(\frac{\beta}{2}\right) = \sqrt{\frac{1-\cos\beta}{2}}$

$= \sqrt{\frac{1-\frac{5}{13}}{2}}$

$= \boxed{\sqrt{\frac{8}{13}}}$

6) $\cos\left(\frac{\beta}{2}\right)$

$\cos\frac{\beta}{2} = \sqrt{\frac{1+\cos\beta}{2}}$

$= \sqrt{\frac{1+\frac{5}{13}}{2}}$

$= \boxed{\sqrt{\frac{18}{26}}}$

7) $\sin\left(\frac{\alpha}{2}\right)$

$\sin\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1-\cos\alpha}{2}}$

$= \sqrt{\frac{1-\frac{12}{13}}{2}}$

$= \sqrt{\frac{1}{26}} = \boxed{\frac{\sqrt{26}}{26}}$