(PART 2) THE DOUBLE-ANGLE FORMULAS

OBJECTIVES: 1) Use the double angle formulas to verify a trig identity.

THE HALF-ANGLE FORMULAS

$$\sin\frac{s}{2} = \pm\sqrt{\frac{1-\cos s}{2}} \qquad \cos\frac{s}{2} = \pm\sqrt{\frac{1+\cos s}{2}} \qquad \tan\frac{s}{2} = \frac{\sin s}{1+\cos s}$$

* The above formulas are proven in Part 6 of your Prove It Notes.

USING HALF ANGLE FORMULAS

1) Express $\cos^4\theta$ in a form that does not involve powers of the trig functions.

$$\cos^4\theta = (\cos^2\theta)^2 = \left(\frac{1+\cos 2\theta}{2}\right)^2 = \frac{1+2\cos 2\theta + \cos^2 2\theta}{4}$$

$$= \frac{1+2\cos 2\theta + \left(\frac{1+\cos 4\theta}{2}\right)}{4} = \frac{2+4\cos 2\theta + 1 + \cos 4\theta}{8}$$

$$= \boxed{\frac{3+4\cos 2\theta + \cos 4\theta}{8}}$$

2) Evaluate $\cos 105°$ using a half-angle formula.

$$\cos 105 = \cos\left(\frac{210}{2}\right) = \pm\sqrt{\frac{1+\cos 210}{2}}$$

$$= \pm\sqrt{\frac{1+\frac{-\sqrt{3}}{2}}{2}} = \pm\sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}} = \pm\sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\pm\sqrt{2-\sqrt{3}}}{2}$$

Since $105°$ is in Q2, $\cos 105° = \boxed{-\frac{\sqrt{2-\sqrt{3}}}{2}}$

3) Express $\sin 2\theta$ and $\cos 2\theta$ in terms of x if $x+1 = 3\sin\theta$ with $\frac{\pi}{2} < \theta < \pi$.

$$x+1 = 3\sin\theta$$

$$\sin\theta = \frac{x+1}{3}$$

$$\cos 2\theta = 1-2\sin^2\theta$$

$$= 1-2\left(\frac{x+1}{3}\right)^2$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$= 2\left(\frac{x+1}{3}\right)\sqrt{1-\sin^2\theta}$$

negative since $\cos\theta = -\#$

$$= -2\left(\frac{x+1}{3}\right)\sqrt{1-\left(\frac{x+1}{3}\right)^2}$$

VERIFYING TRIG IDENTITIES

4) $\frac{\sin 2\theta}{\sin\theta} - \frac{\cos 2\theta}{\cos\theta} = \sec\theta$

LHS.

$\frac{2\sin\theta\cos\theta}{\sin\theta} - \frac{2\cos^2\theta - 1}{\cos\theta} =$

$2\cos\theta - \frac{2\cos^2\theta}{\cos\theta} + \frac{1}{\cos\theta} =$

$2\cos\theta - 2\cos\theta + \frac{1}{\cos\theta} =$

$\frac{1}{\cos\theta} =$

$\sec\theta = \sec\theta$ ✓

5) $\sin 3\theta = 3\sin\theta\cos^2\theta - \sin^3\theta$

L.H.S.

$\sin(\theta + 2\theta) =$

$\sin\theta\cos 2\theta + \cos\theta\sin 2\theta =$

$\sin\theta(\cos^2\theta - \sin^2\theta) + \cos\theta(2\sin\theta\cos\theta) =$

$\sin\theta\cos^2\theta - \sin^3\theta + 2\sin\theta\cos^2\theta =$

$3\sin\theta\cos^2\theta - \sin^3\theta =$ ✓

6) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\cos(\theta + 2\theta) = \cos\theta\cos 2\theta - \sin\theta\sin 2\theta$

$= \cos\theta(\cos^2\theta - \sin^2\theta) - \sin\theta(2\sin\theta\cos\theta)$

$= \cos^3\theta - \sin^2\theta\cos\theta - 2\sin^2\theta\cos\theta$

$= \cos^3\theta - 3\sin^2\theta\cos\theta$

$= \cos^3\theta - 3((1 - \cos^2\theta)\cos\theta)$

$= \cos^3\theta - 3(\cos\theta - \cos^3\theta)$

$= \cos^3\theta - 3\cos\theta + 3\cos^3\theta$

$= 4\cos^3\theta - 3\cos\theta$ ✓

7) $\cos\theta\cos 2\theta\cos 4\theta = \frac{\sin 8\theta}{8\sin\theta}$

$= \frac{\sin(4\theta + 4\theta)}{8\sin\theta} = \frac{\sin 4\theta\cos 4\theta}{4\sin\theta}$

$= \frac{\sin 2\theta\cos 2\theta\cos 4\theta}{2\sin\theta}$

$= \frac{\cancel{\sin\theta}\cos\theta\cos 2\theta\cos 4\theta}{\cancel{\sin\theta}}$

$= \cos\theta\cos 2\theta\cos 4\theta$ ✓