# (PART 1) SUM-TO-PRODUCT FORMULAS

**OBJECTIVES:** 1) Use the sum-to-product/product-to-sum formulas to rewrite an expression.

## THE SUM-TO-PRODUCT/PRODUCT-TO-SUM FORMULAS

$\sin(A+B)+\sin(A-B)=2\sin A\cos B$

$\cos(A+B)+\cos(A-B)=2\cos A\cos B$

$\sin(A+B)-\sin(A-B)=2\cos A\sin B$

$\cos(A+B)-\cos(A-B)=-2\sin A\sin B$

*These formulas are proven in Part 8 of your Prove It Notes.

## USING THE SUM TO PRODUCT FORMULAS

1) Rewrite the product, $\sin 4\theta \sin 3\theta$, as a sum/difference.

use $\cos(A+B)-\cos(A-B)=-2\sin A\sin B$

$\sin 4\theta \sin 3\theta = -\frac{1}{2}\left(\cos(4\theta+3\theta)-\cos(4\theta-3\theta)\right)$

$= -\frac{1}{2}\left(\cos 7\theta - \cos\theta\right)$

2) Rewrite the product, $\cos 3\theta \cos 5\theta$, as a sum/difference.

use $\cos(A+B)+\cos(A-B)=2\cos A\cos B$

$\frac{1}{2}\left(\cos(3\theta+5\theta)+\cos(3\theta-5\theta)\right)=\cos 3\theta\cos 5\theta$

$\frac{1}{2}\left(\cos(8\theta)+\cos(-2\theta)\right)$

$\frac{1}{2}\left(\cos(8\theta)+\cos(2\theta)\right)$

OR!

$\cos 5\theta\cos 3\theta = \frac{1}{2}(\cos 8\theta + \cos 2\theta)$

3) Rewrite the product, $\cos 2\theta \sin 6\theta$, as a sum/difference.

$\sin 6\theta \cos 2\theta$

$\sin(A+B)+\sin(A-B)=2\sin A\cos B$

$\frac{1}{2}\left(\sin(6\theta+2\theta)+\sin(6\theta-2\theta)\right)=\sin 6\theta\cos 2\theta$

$\frac{1}{2}\left(\sin 8\theta+\sin 4\theta\right)=\sin 6\theta \sin 2\theta$

or

$\sin(A+B)-\sin(A-B)=2\cos A\sin B$

$\frac{1}{2}\left(\sin(2\theta+6\theta)-\sin(2\theta-6\theta)\right)=\cos 2\theta\sin 6\theta$

$\frac{1}{2}\left(\sin 8\theta-\sin(-4\theta)\right)=\cos 2\theta\sin 6\theta$

$\frac{1}{2}\left(\sin 8\theta+\sin 4\theta\right)=\cos 2\theta\sin 6\theta$

4) Rewrite the sum, $\cos 11\theta + \cos 3\theta$, as product.

use $\cos(A+B) + \cos(A-B) = 2\cos A \cos B$

$(A+B) + (A-B) = 2A$

$(A+B) - (A-B) = 2B$

$\boxed{\cos 11\theta + \cos 3\theta = 2\cos 7\theta \cos 4\theta}$

$11\theta + 3\theta = 2A$ $\qquad$ $11\theta - 3\theta = 2B$

$14\theta = 2A$ $\qquad$ $8\theta = 2B$

$A = 7\theta$ $\qquad$ $B = 4\theta$

5) Rewrite the difference, $\sin 8\theta - \sin 2\theta$, as a product.

use $\sin(A+B) - \sin(A-B) = 2\cos A \sin B$

$8\theta + 2\theta = 2A$

$A = 5\theta$

$\boxed{\sin 8\theta - \sin 2\theta = 2\cos 5\theta \sin 3\theta}$

$8\theta - 2\theta = 2B$

$B = 3\theta$

6) Find the exact value of $\cos\frac{5\pi}{12}\sin\frac{\pi}{12}$.

use $\sin(A+B) - \sin(A-B) = 2\cos A \sin B$

$$\cos\frac{5\pi}{12}\sin\frac{\pi}{12} = \frac{1}{2}\left(\sin\frac{\pi}{2} - \sin\frac{\pi}{3}\right)$$

$$= \frac{1}{2}\left(1 - \frac{\sqrt{3}}{2}\right) = \boxed{\frac{1}{2} - \frac{\sqrt{3}}{4}}$$

7) Find the exact value of $\sin 105^\circ \sin^\circ 15$.

use

$\cos(A+B) - \cos(A-B) = -2\sin A \sin B$

$$\sin 105 \sin 15 = \frac{-1}{2}\left(\cos 120^\circ - \cos 90^\circ\right)$$

$$= \frac{-1}{2}\left(-\frac{1}{2} - 0\right)$$

$$= \boxed{\frac{1}{4}}$$