9.4
Notes

(PART 1) SOLVING TRIG EQUATIONS

OBJECTIVES: 1) Determine whether a number is a solution to a trig equation.
2) Solve a trig equation by factoring or using identities.
3) Solve a trig equation using your calculator.

IDENTITIES VS. CONDITIONAL EQUATIONS

1) Determine if the following are solutions to the equation $\cos x + \sin x = 1$.

a) $\frac{\pi}{4}$ $\cos\frac{\pi}{4} + \sin\frac{\pi}{4} = 1$

$\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \neq 1$

b) $\frac{\pi}{2}$ $\cos\frac{\pi}{2} + \sin\frac{\pi}{2} = 1$

$0 + 1 = 1$ ✓

Conditional Equation

2) Determine if the following are solutions to the equation $\cos^2 x + \sin^2 x = 1$.

a) $\frac{\pi}{4}$ $\cos^2\frac{\pi}{4} + \sin^2\frac{\pi}{4} = 1$

$\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = 1$ ✓

b) $\frac{\pi}{2}$ Yes!

Identity

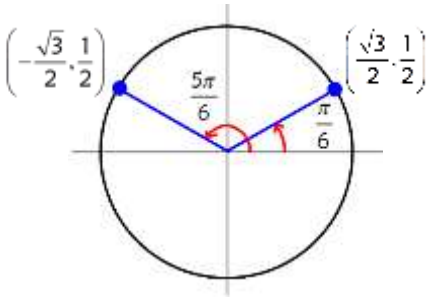
SOLVING A TRIG EQUATION

3) Consider the equation $2\sin x = 1$.

a) Solve for x on the open interval $(0, 2\pi)$.

$\sin x = \frac{1}{2}$

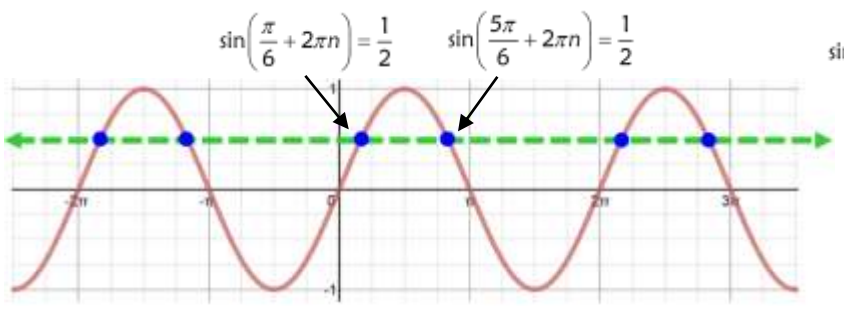
$x = \frac{\pi}{6}$ $x = \frac{5\pi}{6}$

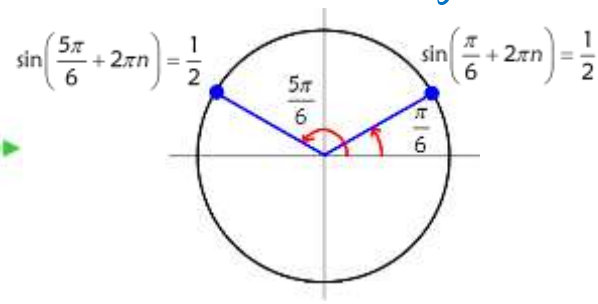


b) Find **all** real-number solutions of the equation.

$x = \frac{\pi}{6} + 2\pi n$ $x = \frac{5\pi}{6} + 2\pi n$

Coterminal Angles!





INVERSE TRIG FUNCTIONS AND YOUR CALCULATOR

c) Use your calculator to find all solutions in the open interval $(0, 2\pi)$.

$\sin x = \frac{1}{2}$

$\sin^{-1}\sin x = \sin^{-1}\left(\frac{1}{2}\right)$

$x \approx .5236$ $\qquad x = \pi - \sin^{-1}\left(\frac{1}{2}\right)$

$x \approx 2.6179$

INVERSE FUNCTIONS

$\cos^{-1}(x)$	inverse cosine
$\sin^{-1}(x)$	inverse sine
$\tan^{-1}(x)$	inverse tangent

SOLVE BY FACTORING

Solve the following equations. Find: a) solutions within the interval $[0, 2\pi)$ and
b) all real-number solutions. Then
c) verify with your calculator.

4) $\tan\theta\cos^2\theta = 2\tan\theta$

$\tan\theta\cos^2\theta - 2\tan\theta = 0$

$\tan\theta(\cos^2\theta - 2) = 0$

$\tan\theta = 0$ $\qquad \cos^2\theta = 2$

$\theta = 0, \pi, \cancel{2\pi}$ $\qquad \cos\theta = \pm\sqrt{2}$

(No solution)

a) $\theta = 0, \pi$

b) $\theta = \pi n$

c) $\tan^{-1}(0) = 0$ $\qquad \cos^{-1}(\pm\sqrt{2}) \rightarrow$ error!

5) $\cos^2\theta + 3\sin\theta = -3$

Quadratic!

$\cos^2\theta = 1 - \sin^2\theta$

$1 - \sin^2\theta + 3\sin\theta + 3 = 0$

$-\sin^2\theta + 3\sin\theta + 4 = 0$

$\sin^2\theta - 3\sin\theta - 4 = 0$

$(\sin\theta + 1)(\sin\theta - 4) = 0$

$\sin\theta = -1$ $\qquad \sin\theta = 4$

$\theta = \frac{3\pi}{2}$ $\qquad$ (No solution)

a) $\theta = \frac{3}{2}\pi$

b) $\theta = \frac{3}{2}\pi + 2\pi n$

$\theta \approx -1.5707 + 2\pi$

$\theta \approx 4.712$

c) $\theta \approx -1.5707 \Rightarrow$ not in $[0, 2\pi)$

EXTRANEOUS SOLUTIONS

6) $\cos x\tan x - \sin x\csc x = 0$

$\sin x - 1 = 0$

$\sin x = 1$

$x = \frac{\pi}{2}$

NO SOLUTION!

$\tan x$ is undefined at $\frac{\pi}{2}$!

7) $\sin t + \cos t = 1 \qquad 0 \le t < 360°$

$\sin^2 t + 2\sin t\cos t + \cos^2 t = 1$

$1 + 2\sin t\cos t = 1$

$2\sin t\cos t = 0$

$\sin t = 0$ $\qquad \cos t = 0$

$t = 0°, 180°, \cancel{360°}$ $\qquad t = 90°, 270°$

$\sin t + \cos t = 1$ $\qquad$ CHECK!

only 0° and 90° work in the original equation!

$t = 0°, 90°$