9.4
Notes

(PART 2) SOLVING TRIG EQUATIONS

OBJECTIVES: 1) Solve a trig equation that involves multiples of an angle.
2) Solve a trig equation by factoring or using identities.

FUNCTIONS INVOLVING MULTIPLES OF AN ANGLE

a) Find ALL solutions to the equation. b) Solutions in the interval $[0, 2\pi]$.

1) $2\sin 2x = \sqrt{3}$

$\sin 2x = \frac{\sqrt{3}}{2}$

$\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{3} + 2\pi n, \quad \frac{2\pi}{3} + 2\pi n$

$2x = \frac{\pi}{3} + 2\pi n \qquad 2x = \frac{2\pi}{3} + 2\pi n$

a) $x = \frac{\pi}{6} + \pi n \qquad x = \frac{\pi}{3} + \pi n$

b) $\left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{\pi}{3}, \frac{4\pi}{3}\right\}$

2) $\tan 3x = \sqrt{3}$

$\tan\theta = \sqrt{3}$

$\theta = \frac{\pi}{3} + \pi n, \quad \frac{4}{3}\pi + \pi n$ ← included here!

$3x = \frac{\pi}{3} + \pi n$

a) $x = \frac{\pi}{9} + \frac{\pi}{3} n \qquad \frac{\pi}{9} + \pi$

b) $\left\{\frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}, \frac{10\pi}{9}, \frac{13\pi}{9}, \frac{16\pi}{9}\right\}$

3) $\cos\frac{x}{2} = \sqrt{2} - \cos\frac{x}{2}$

$2\cos\frac{x}{2} = \sqrt{2}$

$\cos\frac{x}{2} = \frac{\sqrt{2}}{2}$

$\cos\theta = \frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4} + 2\pi n, \quad \frac{7\pi}{4} + 2\pi n$

$\frac{x}{2} = \frac{\pi}{4} + 2\pi n \qquad \frac{x}{2} = \frac{7\pi}{4} + 2\pi n$

a) $x = \frac{\pi}{2} + 4\pi n \qquad x = \frac{7\pi}{2} + 4\pi n$

b) $\left\{\frac{\pi}{2}, \frac{7}{2}\pi\right\}$

4) $4\cos x \sin x = \sqrt{3}$

$2(2\sin x\cos x) = \sqrt{3}$

$\sin 2x = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{3} + 2\pi n, \quad \frac{2}{3}\pi + 2\pi n$

a) $x = \frac{\pi}{6} + \pi n, \quad \frac{\pi}{3} + \pi n$

b) $\left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{\pi}{3}, \frac{4\pi}{3}\right\}$

TRANSFORMING AN EQUATION INTO ONE TRIG FUNCTION

5) $\sqrt{3}\sin\alpha = \cos\alpha$

$\sqrt{3} = \frac{\cos\alpha}{\sin\alpha}$

$\sqrt{3} = \cot\alpha$

$\frac{x}{y}$ $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

a) $\alpha = \frac{\pi}{6} + \pi n$

b) $\left\{\frac{\pi}{6}, \frac{7\pi}{6}\right\}$

6) $\sec^2 x - 2\tan x = 4$

$1 + \tan^2 x - 2\tan x = 4$

$\tan^2 x - 2\tan x - 3 = 0$

$(\tan x - 3)(\tan x + 1) = 0$

$\tan x = 3$ $\tan x = -1$

a) $x = \tan^{-1}(3) + \pi n$ $x = \frac{3\pi}{4} + \pi n$

b) $\left\{1.249, 4.39, \frac{3\pi}{4}, \frac{7\pi}{4}\right\}$

YOU TRY!

7) $\cos^2 x - \sin^2 x = \frac{1}{2}$

$\cos 2x = \frac{1}{2}$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3} + 2\pi n, \frac{5\pi}{3} + 2\pi n$

$2x = \frac{\pi}{3} + 2\pi n$ $2x = \frac{5\pi}{3} + 2\pi n$

a) $x = \frac{\pi}{6} + \pi n$ $x = \frac{5\pi}{6} + \pi n$

b) $\left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}\right\}$

8) $\cos 2x = -\cos x$

$\cos 2x + \cos x = 0$

$2\cos^2 x - 1 + \cos x = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x - 1 = 0$ $\cos x = -1$

$\cos x = \frac{1}{2}$

a) $x = \frac{\pi}{3} + 2\pi n, \frac{5\pi}{3} + 2\pi n, \pi + 2\pi n$

b) $\left\{\frac{\pi}{3}, \frac{5\pi}{3}, \pi\right\}$

USING A CALCULATOR

9) Solve on $[0, 2\pi]$ using a graph (calculator): $\cos(\sin x) = \cos x$

Watch the video!