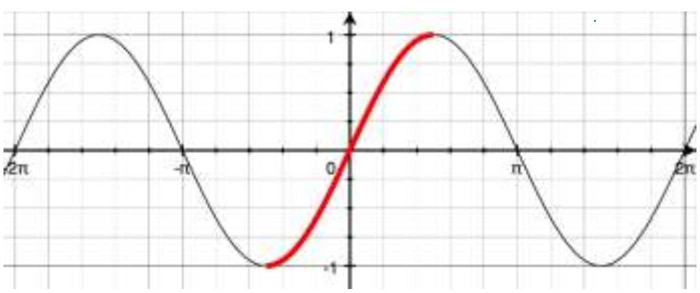
# (PART 1) INVERSE TRIG FUNCTIONS

**OBJECTIVES:** 1) Graph inverse trig functions.
2) Evaluate inverse trig functions and compositions of trig functions.

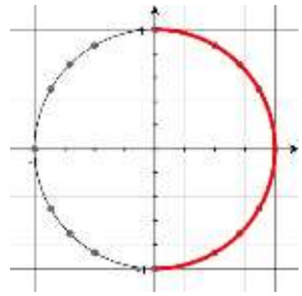
## INVERSE SINE FUNCTION

Recall that a function must be 1 to 1 to have an inverse function. Clearly, the sine function is not one to one, therefore there is no inverse function. However, we consider the restricted sine function, depicted below.




Domain: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

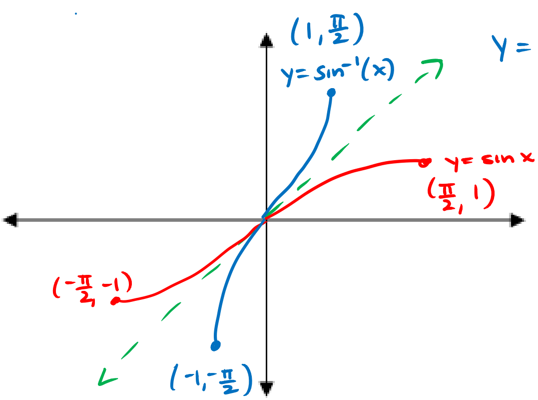
Range: $[-1, 1]$



So, on the restricted domain, $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $y=\sin x$ has a unique inverse function called the **inverse sine function.** Denoted by:

$y = \sin^{-1}(x)$ (consistent with inverse function notation $f^{-1}(x)$)

$y = \arcsin x$ ($\arcsin x$ means the angle (or arc) whose sine is x)




$y = \sin^{-1}(x)$

Domain: $[-1, 1]$

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

1) Evaluate the following:

a) $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

b) $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

c) $\sin^{-1}(-2)$

undefined

$\sin\theta \neq -2$

d) $\sin^{-1}\left(\sin\frac{\pi}{4}\right) = \frac{\pi}{4}$

e) $\arcsin\left(\sin\frac{5\pi}{3}\right) \neq \frac{5\pi}{3}$

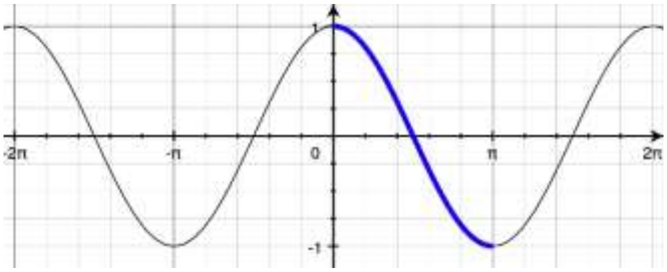
$\sin\frac{5\pi}{3} = \sin -\frac{\pi}{3}$

$\arcsin\left(\sin -\frac{\pi}{3}\right) = \boxed{-\frac{\pi}{3}}$

f) $\sin\left(\sin^{-1}(-2)\right)$

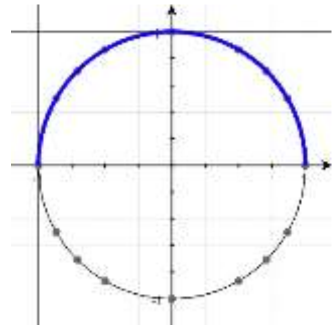
undefined!

## INVERSE COSINE FUNCTION




Domain: $[0,\pi]$

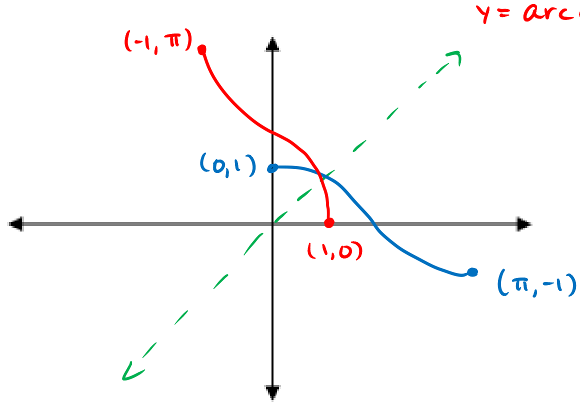
Range: $[-1,1]$



So, on the restricted domain, $[0,\pi]$, $y=\cos x$ has a unique inverse function called the **inverse cosine function.** Denoted by:

$y=\cos^{-1}(x)$

$y=\arccos(x)$




$y=\cos^{-1}(x)$

Domain: $[-1,1]$

Range: $[0,\pi]$

Evaluate the following:

a) $\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$

b) $\cos\left(\cos^{-1}\pi\right)$

undefined!

$\pi$ is not in $[-1,1]$

c) $\arccos\left(\sin\left(\frac{\pi}{3}\right)\right) = \frac{\pi}{6}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

d) $\sin^{-1}\left(\cos\frac{\pi}{3}\right)$

$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$\sin^{-1}\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{6}}$

e) $\arccos\left(\sin\frac{\pi}{4}\right)$

$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

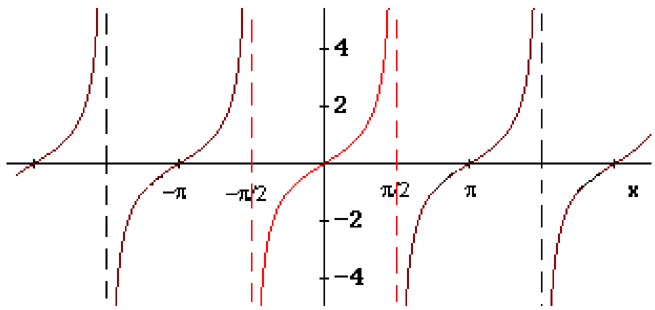
$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

f) $\cos^{-1}\left(\sin\frac{\pi}{6}\right)$

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

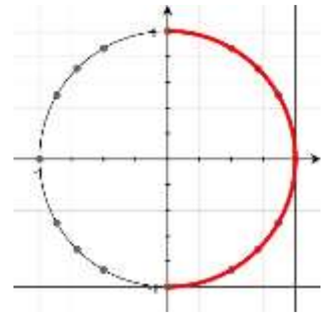
$\cos^{-1}\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{3}}$

# INVERSE TANGENT FUNCTION

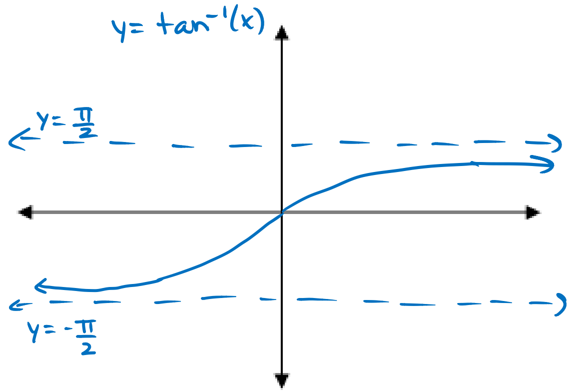



Domain: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Range: $\mathbb{R}$



So, on the restricted domain, $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, $y=\tan x$ has a unique inverse function called the **inverse tangent function.** Denoted by: $y=\tan^{-1}(x)$ $y=\arctan x$




$y=\tan^{-1}(x)$

Domain: $\mathbb{R}$

Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Evaluate the following:

a) $\tan^{-1}(1) = \frac{\pi}{4}$

b) $\arctan\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$

c) $\tan^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$

d) $\csc\left(\tan^{-1}\left(\sqrt{3}\right)\right)$

$\csc\frac{\pi}{3} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$

e) $\tan\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$

$\tan\left(-\frac{\pi}{6}\right) = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \boxed{-\frac{1}{\sqrt{3}}}$ or $-\frac{\sqrt{3}}{3}$

f) $\tan^{-1}\left(\tan\frac{\pi}{3}\right) = \frac{\pi}{3}$

2 ways:

1) You can use $f^{-1}(f(x)) = x$ here because $\frac{\pi}{3}$ is in the range of $\tan^{-1}x$.

2) $\tan\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, so evaluate $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$:

$\tan\theta = \frac{\sqrt{3}}{2}$ when $\theta = \frac{\pi}{3}$.