9.5
Notes

(PART 2) INVERSE TRIG FUNCTIONS

OBJECTIVES: 1) Evaluate compositions of trig functions.
2) Transform a trig expression to an algebraic expression.
3) Use trig identities to help evaluate compositions of trig functions.

COMPOSITION OF FUNCTIONS

1) Evaluate the following:

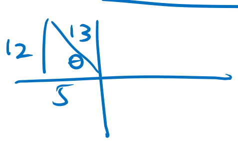
a) $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right) = \frac{5\pi}{6}$

b) $\cos^{-1}\left(\cos\left(\frac{\pi}{6}\right)\right) = \frac{\pi}{6}$

c) $\cos\left(\cos^{-1}\left(\frac{3}{2}\right)\right)$ $\emptyset$

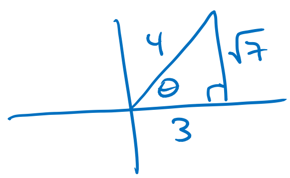
d) $\tan\left(\cos^{-1}\left(-\frac{5}{13}\right)\right) = \frac{-12}{5}$

$\boxed{\tan\theta = -\frac{12}{5}}$



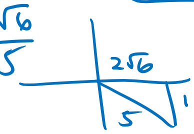
e) $\sin\left(\arccos\frac{3}{4}\right) = \frac{\sqrt{7}}{4}$

$\sin\theta = \frac{\sqrt{7}}{4}$



f) $\cos\left(\arcsin\left(-\frac{1}{5}\right)\right) = \boxed{\frac{2\sqrt{6}}{5}}$

$\cos\theta = \frac{2\sqrt{6}}{5}$



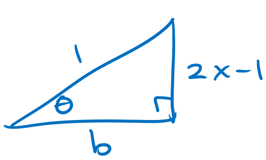
A TOUGH ONE!

2) Find $\sec(\arcsin(2x-1)) = \frac{1}{\sqrt{1-(2x-1)^2}}$

$\sec\theta$

$\sin\theta = \frac{2x-1}{1}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{H}{A}$



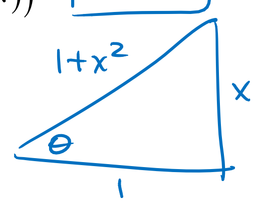
$b^2 + (2x-1)^2 = 1^2$

$b = \sqrt{1-(2x-1)^2}$

3) Find $\sec\left(\tan^{-1}(x)\right) = \boxed{1+x^2}$

$\sec\theta$

$\tan\theta = \frac{x}{1}$



$\sec\theta = \frac{H}{A}$

$\sec\theta = \frac{1+x^2}{1}$

$1^2 + x^2 = c^2$

$c = \sqrt{1+x^2}$

TRANSFORMING A TRIG EXPRESSION

4) If $\tan\theta = \frac{x}{3}$ and $0 < \theta < \frac{\pi}{2}$, express the quantity $\theta - \tan 2\theta$ as a function of x.

$\theta = \tan^{-1}\left(\frac{x}{3}\right)$

$$\theta - \tan 2\theta = \tan^{-1}\left(\frac{x}{3}\right) - \frac{2\tan\theta}{1-\tan^2\theta}$$

$$= \tan^{-1}\left(\frac{x}{3}\right) - \frac{2\left(\frac{x}{3}\right)}{1-\left(\frac{x}{3}\right)^2} = \tan^{-1}\left(\frac{x}{3}\right) - \frac{\frac{2x}{3}}{1-\frac{x^2}{9}}$$

$$= \boxed{\tan^{-1}\frac{x}{3} - \frac{6x}{9-x^2}}$$

5) If $\cos\theta = \frac{5x}{4}$ and $0 < \theta < \frac{\pi}{2}$, express the quantity $2\theta - \cos 2\theta$ as a function of x.

$\theta = \cos^{-1}\left(\frac{5}{4}x\right)$

$$2\theta - \cos 2\theta = 2\left(\cos^{-1}\left(\frac{5x}{4}\right)\right) - \left(2\cos^2\theta - 1\right)$$

$$= 2\left(\cos^{-1}\left(\frac{5x}{4}\right)\right) - \left(2\left(\frac{5x}{4}\right)^2 - 1\right)$$

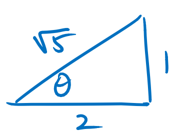
$$= 2\cos^{-1}\left(\frac{5x}{4}\right) - \frac{25x^2}{8} + 1$$

USING AN IDENTITY IN A COMPOSITION

6) $\cos\left(\frac{\pi}{6} + \arctan\frac{1}{2}\right)$

$\theta = \arctan\frac{1}{2}$

$\tan\theta = \frac{1}{2}$



$\sin\theta = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

$\cos\theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$$\cos\left(\frac{\pi}{6} + \theta\right) = \cos\frac{\pi}{6}\cos\theta - \sin\frac{\pi}{6}\sin\theta$$

$$= \frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta$$

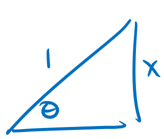
$$= \frac{\sqrt{3}}{2}\left(\frac{2\sqrt{5}}{5}\right) - \frac{1}{2}\left(\frac{\sqrt{5}}{5}\right)$$

$$= \boxed{\frac{2\sqrt{15} - \sqrt{5}}{10}}$$

7) $\cos\left(2\sin^{-1}x\right)$

$\theta = \sin^{-1}(x)$

$\sin\theta = \frac{x}{1}$



$$\cos(2\theta) = 1 - 2\sin^2\theta$$

$$= 1 - 2\left(\frac{x}{1}\right)^2$$

$$= 1 - 2x^2$$

$$\boxed{\cos\left(2\sin^{-1}(x)\right) = 1 - 2x^2}$$