Day 7
In-Class

CHAPTER 11 REVIEW

Name: KEY!

Date: ______________ Period: _______

#1) I CAN SOLVE A SYSTEM OF LINEAR EQUATIONS (W/O A CALC) AND ANY REAL WORLD APPLICATION OF THEM.

a) Sterling Silver is 92.5% pure silver. How many grams of Sterling Silver must be mixed to a 90% Silver alloy to obtain 500 grams of a 91% Silver alloy?

$$\begin{cases} x+y=500 \\ .925x+.90y=500(.91) \end{cases} \Rightarrow \begin{aligned} x+y&=500 \\ 925x+900y&=455000 \\ -925x-925y&=-462500 \\ \hline -25y&=-7500 \\ y&=300 \end{aligned}$$

$x=200$

200 grams of Sterling Silver should be mixed with 90% Silver alloy.

#2) I CAN USE INVERSE MATRICES TO SOLVE A SYSTEM OF EQUATIONS AND SHOW THE APPROPRIATE WORK!

a) Solve the system: $\begin{cases} 3x-9y=2 \\ 9x+18y=1 \end{cases}$

$A=\begin{bmatrix} 3 & -9 \\ 9 & 18 \end{bmatrix}$ $B=\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $A^{-1}B=\begin{bmatrix} \frac{1}{3} \\ -\frac{1}{9} \end{bmatrix}$ $\boxed{\left(\frac{1}{3}, -\frac{1}{9}\right)}$

#3) I CAN USE ROW REDUCED ECHELON FORM TO SOLVE A SYSTEM AND SHOW THE APPROPRIATE WORK!

a) Solve the system: $\begin{cases} 5x-8y=4 \\ 2x-4y=1 \end{cases}$

$A=\begin{bmatrix} 5 & -8 & 4 \\ 2 & -4 & 1 \end{bmatrix}$ $\text{rref } A=\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3/4 \end{bmatrix}$ $\boxed{\left(2, \frac{3}{4}\right)}$

#4) I CAN USE DETERMINANTS AND CRAMER'S RULE TO SOLVE A SYSTEM AND SHOW THE APPROPRIATE WORK!

a) Solve the system: $\begin{cases} x-4y=32 \\ 3x+y=5 \end{cases}$

$D_x=\begin{vmatrix} 32 & -4 \\ 5 & 1 \end{vmatrix}$ $D_y=\begin{vmatrix} 1 & 32 \\ 3 & 5 \end{vmatrix}$

$\det\begin{bmatrix} 1 & -4 \\ 3 & 1 \end{bmatrix}=13$

$x=\frac{D_x}{D}$ $y=\frac{D_y}{D}$ $\boxed{(4, -7)}$

$x=\frac{52}{13}$ $y=\frac{-91}{13}$

#5) I CAN DISCERN BETWEEN A CONSISTENT/INCONSISTENT AND DEPENDENT/INDEPENDENT SYSTEM OF EQUATIONS.

Determine if the system is consistent or inconsistent. If applicable, state whether the system is dependent/independent.

a) $\begin{cases} 8x - 12y = 24 \\ 6x - 9y = 18 \end{cases}$

consistent

dependent

(infinite solutions)

b) $\begin{cases} 11x - 5y = -38 \\ 9x + 2y = -25 \end{cases}$

consistent

independent

(1 unique solution)

c) $\begin{cases} 4x - 6y = 11 \\ 6x - 9y = 18 \end{cases}$

inconsistent

(no solution)

#6) I CAN WRITE THE SOLUTIONS TO A DEPENDENT, CONSISTENT SYSTEM IN TERMS OF ONE VARIABLE

a) Solve the system: $\begin{cases} 3z + y - 1 = 0 \\ x + y = 3 \\ 2y + 3z + x - 4 = 0 \end{cases}$

$A = \begin{bmatrix} 0 & 1 & 3 & 1 \\ 1 & 1 & 0 & 3 \\ 1 & 2 & 3 & 4 \end{bmatrix}$ $\quad$ $\text{rref } A = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

In terms of z:

$y + 3z = 1 \Rightarrow y = -3z + 1$

$x - 3z = 2 \quad x = 3z + 2$

$\boxed{(3z+2, -3z+1, z)}$

#7) I CAN MULTIPLY MATRICES WITHOUT A CALCULATOR AND I KNOW WHEN I CAN'T MULTIPLY MATRICES.

Perform the indicated operation:

a) $\begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ -2 & 5 \end{bmatrix}$ $\quad 2\times2 \cdot 2\times2$

$\begin{bmatrix} 3\cdot4 + -1\cdot-2 & 3\cdot1 + -1\cdot5 \\ 4\cdot4 + -2\cdot-2 & 4\cdot1 + -2\cdot5 \end{bmatrix}$

$\begin{bmatrix} 14 & -2 \\ 20 & -6 \end{bmatrix}$

b) $\begin{bmatrix} x \\ y \\ -z \end{bmatrix}\begin{bmatrix} 2a & b \end{bmatrix}$ $\quad 3\times1 \cdot 1\times2$

$\begin{bmatrix} 2ax & bx \\ 2ay & by \\ -2az & -bz \end{bmatrix}$

#8) I CAN SOLVE A NONLINEAR SYSTEM OF EQUATIONS.

Solve the systems:

a) $\begin{cases} \dfrac{2}{x^2} + \dfrac{3}{y^2} = -16 \\ \dfrac{3}{x^2} - \dfrac{2}{y^2} = 28 \end{cases}$

Let $a = \frac{1}{x^2}$ $b = \frac{1}{y^2}$

$\begin{cases} 2a + 3b = -16 \\ 3a - 2b = 28 \end{cases}$

$A = \begin{bmatrix} 2 & 3 & -16 \\ 3 & -2 & 28 \end{bmatrix}$ rref $A = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -8 \end{bmatrix}$

$a = 4$ $b = -8$

$\frac{1}{x^2} = 4$ $\frac{1}{y^2} = -8$

$x = \pm\frac{1}{2}$ not real!

no real solutions to this system!

b) $\begin{cases} y = e^x \\ y = 3e^{2x} - 2 \end{cases}$ $\leftarrow$ $y = 3(e^x)^2 - 2$

$e^x = y$

$y = 3y^2 - 2$

$3y^2 - y - 2 = 0$

$(3y+2)(y-1) = 0$

$y = \frac{-2}{3}$ $y = 1$

If $y = -\frac{2}{3}$

$-\frac{2}{3} = e^x$

no real solution!

If $y = 1$

$e^x = 1$

$\ln 1 = x$

$x = 0$

Solution: $(0, 1)$

#9) I CAN WRITE PARTIAL FRACTIONS WITH REPEATED LINEAR FACTORS AND IRREDUCIBLE QUADRATIC FACTORS.

Rewrite the fraction as the sum of two simpler fractions:

a) $\dfrac{-3x+11}{x^2-6x+9} = \dfrac{-3x+11}{(x-3)^2} = \dfrac{A}{x-3} + \dfrac{B}{(x-3)^2}$

$-3x + 11 = A(x-3) + B$

let $x = 3$

$-3(3) + 11 = B$

$B = 2$

$-3 = A$

$\boxed{\dfrac{-3}{x-2} + \dfrac{2}{(x-3)^2}}$

b) $\dfrac{-x^2-5x-1}{x^3-2x^2+x-2} = \dfrac{-x^2-5x-1}{x^2(x-2)+1(x-2)}$

$\dfrac{-x^2-5x-1}{(x^2+1)(x-2)} = \dfrac{Ax+B}{x^2+1} + \dfrac{C}{x-2}$

$-x^2-5x-1 = (Ax+B)(x-2) + C(x^2+1)$

let $x = 2$

$-4-10-1 = C(5)$

$-15 = 5C$

$\boxed{C = -3}$

Equate coefficients:

x^2: $-1 = A + C$

$-1 = A - 3$

$\boxed{A = 2}$

constant:

$-1 = -2B + C$

$-1 = -2B - 3$

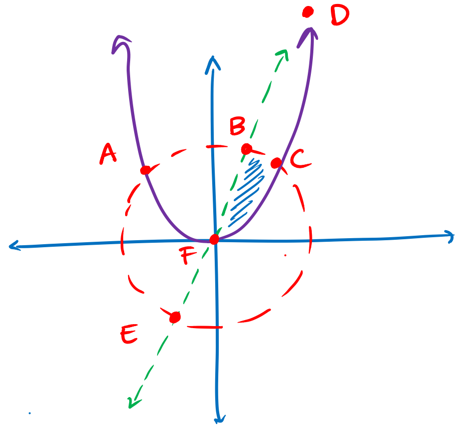
$\boxed{B = -1}$

$\boxed{\dfrac{2x-1}{x^2+1} - \dfrac{3}{x-2}}$

#10) I CAN SOLVE A SYSTEM OF INEQUALITIES.

Graph the solution to the system of inequalities. Make sure to show the points of intersection.

a) $\begin{cases} y - x^2 \geq 0 \\ x^2 + y^2 < 1 \\ y < 2x \end{cases}$



F(0,0)

A & C $\begin{cases} x^2+y^2=1 \\ y=x^2 \end{cases}$

$y + y^2 = 1$

$y^2 + y - 1 = 0$

$y = \dfrac{-1 \pm \sqrt{5}}{2}$ $\quad \cancel{\dfrac{-1-\sqrt{5}}{2}}$

$y = x^2$ ← $y \geq 0$!

let $y = \dfrac{-1+\sqrt{5}}{2}$

$x^2 = \dfrac{-1+\sqrt{5}}{2}$

$x = \pm\sqrt{\dfrac{-1+\sqrt{5}}{2}}$

$A\left(\sqrt{\dfrac{-1+\sqrt{5}}{2}}, \dfrac{-1+\sqrt{5}}{2}\right)$

$C\left(-\sqrt{\dfrac{-1+\sqrt{5}}{2}}, \dfrac{-1+\sqrt{5}}{2}\right)$

B & E $\begin{cases} y = 2x \\ x^2+y^2=1 \end{cases}$

$x^2 + 4x^2 = 1$

$5x^2 = 1$

$x = \pm\sqrt{\frac{1}{5}}$

if $x = \sqrt{\frac{1}{5}}$ $\quad$ if $x = -\sqrt{\frac{1}{5}}$

$y = 2\sqrt{\frac{1}{5}}$ $\quad$ $y = -2\sqrt{\frac{1}{5}}$

$B\left(\sqrt{\frac{1}{5}}, 2\sqrt{\frac{1}{5}}\right)$

$E\left(-\sqrt{\frac{1}{5}}, -2\sqrt{\frac{1}{5}}\right)$

D $\begin{cases} y = 2x \\ y = x^2 \end{cases}$

$x^2 = 2x$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0 \quad x = 2$

$y = 4$

D(2,4)

#11) I CAN USE LINEAR PROGRAMMING TO FIND THE BEST POSSIBLE OUTCOME FOR A GIVEN SET OF CONSTRAINTS.

a) You are taking a test in which items of type A are worth 10 points and items of type B are worth 15 points. It takes 3 minutes to answer each type A question and 6 minutes to answer each type B question. Total time allowed is 60 minutes, and you may not answer more than 16 questions. However, you must answer at least 2 of each type of question. Assuming all of your answers are correct, how many of each type should you answer in order to get the best score?

x = # of type A Qs

y = # of type B Qs

Obj. Quantity:

$10x + 15y$

Constraints:

$3x + 2y \leq 60$

$x + y \leq 16$

$x \geq 2$

$y \geq 2$

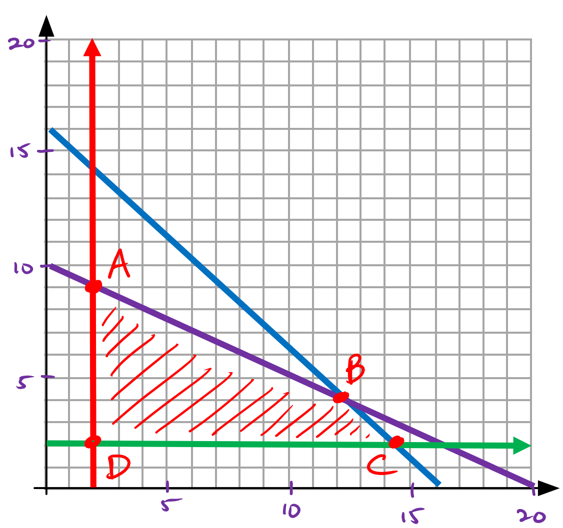
Corner Pts:

$A(2,9)$ – $\begin{cases} 3x+6y=60 \\ x=2 \end{cases}$

$B(12,4)$ – $\begin{cases} 3x+6y=60 \\ x+y=16 \end{cases}$

$C(14,2)$ – $\begin{cases} x+y=16 \\ y=2 \end{cases}$

$D(2,2)$ – $\begin{cases} x=2 \\ y=2 \end{cases}$



Test Corner Pts:

A(2,9) 155pts $\quad$ C(14,2) 170pts

B(12,4) 180pts $\quad$ D(2,2) 50pts

Max of 180 pts when answering 12 type A & 4 type B questions.

- MAKE SURE THESE QUESTIONS SEEM EASY PEASY LEMON SQUEEZY!
- REVIEW YOUR HOMEWORK
- REVIEW YOUR NOTES
- REVIEW UNTIL YOUR BRAIN HURTS!