Day 6
In-Class

CHAPTER 13 REVIEW

Name: KEY

Date: ____________ Period: ______

#1) I CAN USE LONG DIVISION/SYNTHETIC DIVISION TO DIVIDE POLYNOMIALS.

a) $\dfrac{x^4+3x^2+1}{x^2-2x+3}$ $\boxed{x^2+2x+4+\dfrac{2x-11}{x^2-2x+3}}$

$x^2-2x+3 \overline{)\,x^4+0x^3+3x^2+0x+1}$ quotient x^2+2x+4

$-(x^4-2x^3+3x^2)$

$2x^3+0x^2+0x$

$-(2x^3-4x^2+6x)$

$4x^2-6x+1$

$-(4x^2-8x+12)$

$2x-11$

b) $\dfrac{2x^3+4x^2+7x+4}{x+3}$

$-3 \mid 2 \quad 4 \quad 7 \quad 4$

$\quad -6 \quad 6 \quad -39$

$2 \quad -2 \quad 13 \quad -35$

$\boxed{2x^2-2x+13-\dfrac{35}{x+3}}$

#2) I CAN FACTOR POLYNOMIALS OF THE FORM $X^N - A^N$ INTO THE PRODUCT OF (ONLY) TWO FACTORS.

a) x^4-81

$(x-3)$

$3 \mid 1 \quad 0 \quad 0 \quad 0 \quad -81$

$\quad 3 \quad 9 \quad 27 \quad 81$

$1 \quad 3 \quad 9 \quad 27 \quad 0$

$\boxed{(x-3)(x^3+3x^2+9x+27)}$

#3) I CAN USE THE REMAINDER THEOREM TO EVALUATE A FUNCTION AT A GIVEN VALUE OF X.

a) $4x^3-7x^2+8x+2$ at $x=2$

$2 \mid 4 \quad -7 \quad 8 \quad 2$

$\quad 8 \quad 2 \quad 20$

$4 \quad 1 \quad 10 \quad 22$

$\boxed{f(2)=22}$

b) $3x^4-5x^3+3x^2-2x+5$ at $x=-3$

$-3 \mid 3 \quad -5 \quad 3 \quad -2 \quad 5$

$\quad -9 \quad 42 \quad -135 \quad 411$

$3 \quad -14 \quad 45 \quad -137 \quad 416$

$\boxed{f(-3)=416}$

#5) I CAN USE THE FACTOR THEOREM TO SOLVE A POLYNOMIAL EQUATION OR WRITE ONE.

a) Solve the equation $x^3-3x^2+12x-10=0$ given that one root is $1+3i$.

$r_1=1+3i \quad r_2=1-3i$

$x^2-(r_1+r_2)x+r_1r_2=0$

$b=-2 \quad c=10$

$x^2-2x+10$

$x^2-2x+10 \overline{)\,x^3-3x^2+12x-10}$ quotient $x-1$

$-(x^3-2x^2+10x)$

$-x^2+2x-10$

$-(-x^2+2x-10)$

0

$\boxed{x=1\pm 3i,\ 1}$

b) Find the equation of a polynomial function of least degree with integer coefficients with the following roots: -4 of multiplicity 2 and $-2i$.

$(x+4)^2(x+2i)(x-2i)=0$

x^2+4

$(x+4)^2=x^2+8x+16$

$(x^2+4)(x^2+8x+16)=x^4+8x^3+16x^2+4x^2+32x+64$

$\boxed{f(x)=x^4+8x^3+20x^2+32x+64}$

#6) I CAN (QUICKLY) WRITE A QUADRATIC EQUATION WHEN GIVEN ITS ROOTS.

a) Write the quadratic equation with a root of $r_1 = 3 - 2i$.

$\boxed{x^2 - 6x + 13 = 0}$

#7) I CAN USE COMPLEX NUMBERS TO WRITE THE QUOTIENT OF TWO COMPLEX NUMBERS IN STANDARD FORM.

a) $\dfrac{4-2i}{3i} \cdot \dfrac{i}{i} = \dfrac{4i-2i^2}{-3} = \dfrac{4i+2}{-3}$

$\boxed{\dfrac{2+4i}{-3}}$

b) $\dfrac{5+3i}{7-2i} \cdot \dfrac{7+2i}{7+2i}$

$\dfrac{35+10i+21i+6i^2}{49-4i^2} = \boxed{\dfrac{29+31i}{53}}$

#8) I CAN PERFORM OPERATIONS ON COMPLEX NUMBERS AND SIMPLIFY POWERS OF i.

a) $(3-2i)(1+4i)$

$3+12i-2i-8i^2$

$\boxed{11+10i}$

b) i^{25}

$=(i^2)^{12} i$

$=\boxed{i}$

c) i^{39}

$=(i^2)^{19} i$

$=\boxed{-i}$

d) i^{3002}

$=(i^2)^{1501}$

$=\boxed{-1}$

#9) I CAN USE THE RATIONAL ROOTS THEOREM TO LIST ALL POSSIBLE RATIONAL ROOTS OF A POLYNOMIAL.

a) List all possible rational zeros of the function $f(x) = 6x^3 - 8x^2 - 9x + 12$.

$\dfrac{p}{q} = \dfrac{\pm 1, 2, 3, 4, 6, 12}{\pm 1, 2, 3, 6} = \boxed{\pm 1, \tfrac{1}{2}, \tfrac{1}{3}, \tfrac{1}{6}, 2, \tfrac{2}{3}, 3, \tfrac{3}{2}, 4, \tfrac{4}{3}, 6, 12}$

#10) I CAN USE DECARTE'S RULE OF SIGNS TO PREDICT ALL POSSIBLE COMBINATIONS OF ROOTS.

List all possible combinations of roots:

a) $f(x) = 5x^4 - 7x^3 + 8x^2 + 4x - 3$

Max 3 positive roots

$f(-x) = 5x^4 + 7x^3 + 8x^2 - 4x - 3$

Max 1 neg. root

+	−	i
3	1	0
1	1	2

b) $f(x) = 2x^5 + x^4 - 3x^3 - 2x^2 - 4x + 5$

Max 2 positive roots

$f(-x) = -2x^5 + x^4 + 3x^3 - 2x^2 + 4x + 5$

Max 3 neg. roots

+	−	i
2	3	0
0	3	2
2	1	2
0	1	4

#11) I CAN USE THE UPPER AND LOWER BOUND THEOREM TO FIND NARROW DOWN ZEROS OF POLYNOMIALS.

Verify the upper and lower bounds:

a) $f(x) = x^3 + 5x^2 + 3x - 2$ upper bound: 1 lower bound: –6

$$\begin{array}{r|rrrr} 1 & 1 & 5 & 3 & -2 \\ & & 1 & 6 & 9 \\ \hline & 1 & 6 & 9 & 7 \end{array} \qquad \begin{array}{r|rrrr} -6 & 1 & 5 & 3 & -2 \\ & & -6 & 6 & -54 \\ \hline & 1 & -1 & 9 & -56 \end{array}$$

all positive, ∴ 1 is an upperbound.

alternating signs, ∴ –6 is a lower bound.

#12) I CAN FIND ZEROS OF POLYNOMIALS BY FACTORING.

a) $f(x) = x^4 + 2x^2 - 8$

$x^4 + 4x^2 - 2x^2 - 8 = 0$

$x^2(x^2+4) - 2(x^2+4) = 0$

$(x^2-2)(x^2+4) = 0$

$\boxed{x = \pm 2, \pm 2i}$

b) $f(x) = x^4 - 4x^3 + 3x^2 + 8x - 16$ if one factor is $x^2 - x - 4$

$$\begin{array}{r|l} & x^2 - 3x + 4 \\ x^2 - x - 4 & x^4 - 4x^3 + 3x^2 + 8x - 16 \\ & -(x^4 - x^3 - 4x^2) \\ & -3x^3 + 7x^2 + 8x \\ & -(-3x^3 + 3x^2 + 12x) \\ & 4x^2 - 4x - 16 \\ & -(4x^2 - 4x - 16) \\ & 0 \end{array}$$

$x^2 - 3x + 4 = 0$

$\frac{3 \pm \sqrt{9-4(4)}}{2}$

$x = \frac{3 \pm \sqrt{-7}}{2}$

$x = \frac{3 \pm i\sqrt{7}}{2}$

$x^2 - x - 4 = 0$

$x = \frac{1 \pm \sqrt{1-4(-4)}}{2}$

$x = \frac{1 \pm \sqrt{17}}{2}$

$\boxed{\text{Solutions: } \frac{1 \pm \sqrt{17}}{2}, \frac{3 \pm i\sqrt{7}}{2}}$

#13) I CAN FIND ZEROS OF POLYNOMIALS WHEN GIVEN A FACTOR OF THE POLYNOMIAL.

a) Find all zeros of the polynomial $f(x) = x^4 + 2x^3 - 3x^2 + 2x - 4$ if one factor of $f(x)$ is $(x - i)$.

$\pm i$ roots, ∴ $x^2 + 1$ is a factor

$$\begin{array}{r|l} & x^2 + 2x - 4 \\ x^2 + 0x + 1 & x^4 + 2x^3 - 3x^2 + 2x - 4 \\ & -(x^4 + 0x^3 + x^2) \\ & 2x^3 - 4x^2 + 2x \\ & -(2x^3 + 0x^2 + 2x) \\ & -4x^2 - 4 \\ & -(-4x^2 - 4) \\ & 0 \end{array}$$

$x^2 + 2x - 4 = 0$

$x = \frac{-2 \pm \sqrt{4-4(-4)}}{2}$

$x = \frac{-2 \pm \sqrt{20}}{2}$

$x = -1 \pm \sqrt{5}$

$\boxed{\text{Zeros: } \pm i, -1 \pm \sqrt{5}}$