Day 9
In-Class

CHAPTER 5 REVIEW

Name: KEY

Date: _____ Period: _____

#1) I CAN GRAPH EXP. AND LOG FUNCTIONS USING TRANSFORMATIONS AND DETERMINE THEIR DOMAIN AND RANGE.

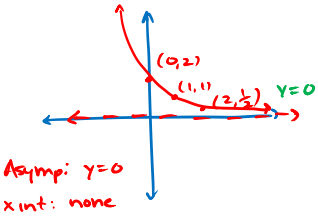
Sketch the graph. Show all important information (intercepts, asymptotes, domain/range).

a) $y = 2^{-x+1}$

D: $(-\infty,\infty)$
R: $(0,\infty)$

$2^{-(x-1)}$ (F_y, R_1)

$(-1,\frac{1}{2}) \to (1,\frac{1}{2})\ (2,\frac{1}{2})$
$(0,1) \to (0,1)\ (1,1)$
$(1,2) \to (-1,2)\ (0,2)$

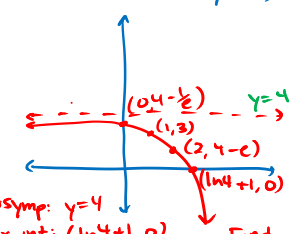


Asymp: $y=0$
x int: none
y int: $(0,2)$

b) $y = -e^{x-1} + 4$

D: $(-\infty,\infty)$ (F_x, R_1, U_4)
R: $(-\infty,4)$

$(-1,\frac{1}{e}) \to (0,-\frac{1}{e})\ (0,4-\frac{1}{e})$
$(0,1) \to (1,-1)\ (1,3)$
$(1,e) \to (2,-e)\ (2,4-e)$



Asymp: $y=4$
x int: $(\ln 4+1, 0)$
y int: $(0, 4-\frac{1}{e})$

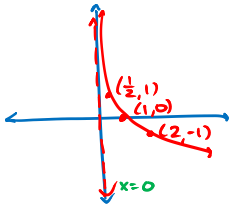
Find x int:
$0 = -e^{x-1}+4$
$e^{x-1} = 4$
$\ln 4 = x-1$
$\ln 4+1 = x$

c) $y = -\log_2(x)$

D: $(0,\infty)$
R: $(-\infty,\infty)$

mother: 2^x (F_x)

$(-1,\frac{1}{2}) \to (\frac{1}{2},-1)\ (\frac{1}{2},1)$
$(0,1) \to (1,0)\ (1,0)$
$(1,2) \to (2,1)\ (2,-1)$



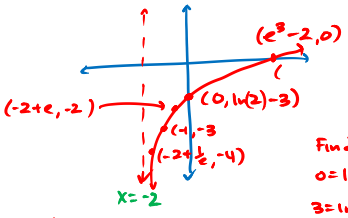
Asymp: $x=0$
x int: $(1,0)$
y int: none

d) $y = \ln(x+2) - 3$

D: $(-2,\infty)$
R: $(-\infty,\infty)$

mother: e^x (L_2, D_3)

$(-1,\frac{1}{e}) \to (\frac{1}{e},-1)\ (-2+\frac{1}{e},-4)$
$(0,1) \to (1,0)\ (-1,-3)$
$(1,e) \to (e,1)\ (-2+e,-2)$



Asymp: $x=-2$
x int: $(e^3-2, 0)$
y int: $(0, \ln 2-3)$

Find x int:
$0 = \ln(x+2)-3$
$3 = \ln(x+2)$
$e^3 = x+2$
$e^3-2 = x$

Find y int:
$y = \ln(2)-3$

#2) I CAN EVALUATE A VARIETY OF LOGARITHMIC AND EXPONENTIAL EXPRESSIONS.

Evaluate the following:

a) $\log_5 0.2$

$\log_5 \frac{1}{5}$
$\log_5 1 - \log_5 5$
$0-1$
-1

b) $\log_{\frac{1}{2}} 64$

$\log_{\frac{1}{2}} 2^6 = ?$
$(2^{-1})^? = 2^6$
-6

c) $3\log 1$

$3 \cdot 0$
0

d) $\left(\frac{27}{16}\right)^{-\frac{2}{3}}$

$\left(\frac{16}{27}\right)^{\frac{2}{3}}$
$\frac{(2^4)^{2/3}}{(3^3)^{2/3}} = \frac{2^{8/3}}{9} = \frac{\sqrt[3]{256}}{9}$

#3) I CAN THINK OUTSIDE THE BOX!

Given $\log_b 2 = A$, $\log_b 5 = B$, and $\log_b 7 = C$, evaluate the following:

a) $\log_b 70$

$\log_b 2 \cdot 5 \cdot 7$
$\log_b 2 + \log_b 5 + \log_b 7$
$A+B+C$

b) $\log_b \left(\sqrt[3]{5}\right)$

$\log_b 5^{\frac{1}{3}}$
$\frac{1}{3}\log_b 5$
$\frac{1}{3}B$

c) $\log_b \sqrt{2.5}$

$\log_b \left(\frac{5}{2}\right)^{\frac{1}{2}}$
$\frac{1}{2}\log_b \frac{5}{2}$
$\frac{1}{2}(\log_b 5 - \log_b 2)$
$\frac{1}{2}(B-A)$

#4) I CAN SOLVE A VARIETY OF EXPONENTIAL EQUATIONS AND INEQUALITIES.

Solve for x:

a) $x^{-\frac{3}{2}} = 27$

$\left(x^{-\frac{3}{2}}\right)^{-\frac{2}{3}} = \left(3^3\right)^{-\frac{2}{3}}$
$x = 3^{-2}$
$x = \frac{1}{9}$

b) $\left(\frac{1}{9}\right)^{2x-5} = 81^{x+3}$

$\left(3^{-2}\right)^{2x-5} = \left(3^4\right)^{x+3}$
$-4x+10 = 4x+12$
$8x = -2$
$x = -\frac{1}{4}$

c) $4^{6x+2} = 9^{5x-3}$

$\ln 4^{6x+2} = \ln 9^{5x-3}$
$(6x+2)\ln 4 = (5x-3)\ln 9$
$6x\ln 4 + 2\ln 4 = 5x\ln 9 - 3\ln 9$
$6x\ln 4 - 5x\ln 9 = -2\ln 4 - 3\ln 9$
$x(6\ln 4 - 5\ln 9) = -2\ln 4 - 3\ln 9$
$x = \frac{-2\ln 4 - 3\ln 9}{6\ln 4 - 5\ln 9}$

d) $5 < e^{3x-2} < 8$

$\ln 5 < \ln e^{3x-2} < \ln 8$
$\ln 5 < 3x-2 < \ln 8$
$\ln 5 + 2 < 3x < \ln 8 + 2$
$\frac{2+\ln 5}{3} < x < \frac{2+\ln 8}{3}$

Interval notation: $\left(\frac{2+\ln 5}{3}, \frac{2+\ln 8}{3}\right)$

e) $5^{3x-4} < e^{7x-2}$

$\ln 5^{3x-4} < \ln e^{7x-2}$

$(3x-4)\ln 5 < 7x-2$

$3x\ln 5 - 4\ln 5 < 7x-2$

$3x\ln 5 - 7x < 4\ln 5 - 2$

$x(3\ln 5 - 7) < 4\ln 5 - 2$

$x > \dfrac{4\ln 5 - 2}{3\ln 5 - 7}$ ($3\ln 5 - 7 \approx 5$)

Flip the sign bk $3\ln 5 - 7$ is negative!

f) $2^{x+4} = 3e^{4x+5}$

$\ln 2^{x+4} = \ln 3 + 4x + 5$

$(x+4)\ln 2 = \ln 3 + 4x + 5$

$x\ln 2 + 4\ln 2 = \ln 3 + 4x + 5$

$x\ln 2 - 4x = 5 + \ln 3 - 4\ln 2$

$x(\ln 2 - 4) = 5 + \ln 3 - 4\ln 2$

$x = \dfrac{5 + \ln 3 - 4\ln 2}{-4 + \ln 2}$

g) $9 = \dfrac{20}{1 - e^{-2x}}$

$9(1 - e^{-2x}) = 20$

$-9e^{-2x} = 20 - 9$

$-9e^{-2x} = 11$

$e^{-2x} = \dfrac{-11}{9}$

$-2x = \ln\left(\dfrac{-11}{9}\right)$

not real no solution!

h) $e^{2x} - 5e^{x} + 6 = 0$

let $t = e^x$

$t^2 - 5t + 6 = 0$

$(t-3)(t-2) = 0$

$t = 3, 2$

$e^x = 3 \quad e^x = 2$

$x = \ln 3 \quad x = \ln 2$

#5) I CAN SOLVE A VARIETY OF LOGARITHMIC EQUATIONS AND INEQUALITIES.

Solve for x:

a) $x = \ln e^5$

$x = 5$

b) $\ln x = 3e$

$x = e^{3e}$

c) $\log_{10}\left[\log_2\left(\log_7 x\right)\right] = 0$

$\log_2(\log_7 x) = 1$

$\log_7 x = 2$

$x = 49$

d) $10^{\log(5x)} = 3$

$5x = 3$

$x = \dfrac{3}{5}$

e) $\log_2(x+3) - 1 = \log_2(x-1)$

$\log_2(x+3) - \log_2(x-1) = 1$

$\log_2 \dfrac{(x+3)}{(x-1)} = 1$

$\dfrac{x+3}{x-1} = 2$

$x+3 = 2(x-1)$

$x+3 = 2x-2$

$x = 5$

f) $\log_6(x-1) + \log_6(x+2) = \dfrac{1}{3}\log_5 125$ ($x > 1$, $x > -2$)

Domain: $x > 1$

$\log_6(x-1)(x+2) = \dfrac{1}{3} \cdot 3$

$\log_6(x^2 + x - 2) = 1$

$x^2 + x - 2 = 6$

$x^2 + x - 8 = 0$

$x = \dfrac{-1 \pm \sqrt{1-4(-8)}}{2} = \dfrac{-1 \pm \sqrt{33}}{2}$

$x = \dfrac{-1 + \sqrt{33}}{2}$

$\dfrac{-1 - \sqrt{33}}{2}$

g) $\dfrac{\log(2x+3)}{\log(x)} = \log_4 16$

Domain: $x > 0$

$\dfrac{\log(2x+3)}{\log(x)} = 2$

$\log(2x+3) = 2\log(x)$

$\log(2x+3) = \log x^2$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3, -1$

$x = 3$

#6) I CAN SOLVE A VARIETY OF EXPONENTIAL & LOGARITHMIC APPLICATIONS. (USE A CALCULATOR)

a) Easy: $15,000 is deposited into an account that pays 6.25% annual interest, compounded weekly. Find the balance after 7 years.

$A = P\left(1 + \dfrac{r}{n}\right)^{nt}$

$A = 15000\left(1 + \dfrac{.0625}{52}\right)^{52(7)}$ $\quad A = 23,226.35$ $\quad \$23,226.35$

b) How much must you deposit into an account that pays 5.75% interest, compounded monthly, to have a balance of $20,000 after 5 years?

$A = P\left(1 + \dfrac{r}{n}\right)^{nt}$

$20,000 = P\left(1 + \dfrac{.0575}{12}\right)^{12 \cdot 5}$ $\quad P = \dfrac{20,000}{\left(1 + \dfrac{.0525}{12}\right)^{60}}$ $\quad P = 15,013.03589$

$\$15,013.04$ ← rd up bk you need at least this much

c) At what interest rate must a sum of $5000 be deposited in order for it to become $7500 in 5 years if interest is compounded quarterly?

$7500 = 5000\left(1 + \dfrac{r}{4}\right)^{4 \cdot 5}$

$\dfrac{3}{2} = \left(1 + \dfrac{r}{4}\right)^{20}$

$\sqrt[20]{\dfrac{3}{2}} = 1 + \dfrac{r}{4}$

$-1 + \sqrt[20]{\dfrac{3}{2}} = \dfrac{r}{4}$

$r = 4\left(-1 + \sqrt[20]{\dfrac{3}{2}}\right)$

$\approx .0819$

8.19%

d) How long will it take a savings account of $2000 to have a value of $15,000 if the interest rate is 7.5% and is compounded continuously?

$t = Pe^{rt}$

$15000 = 2000e^{.075t}$

$\frac{15}{2} = e^{.075t}$

$\ln \frac{15}{2} = .075t$

$\frac{\ln 15/2}{.075} = t$

$t \approx 27$ yrs (26.865...)

e) The population of a single-celled organism decreased from 480 to 58 in 12 minutes. What is the half-life of this organism? Find k first!

$58 = 480e^{k \cdot 12}$

$\frac{58}{480} = e^{12k}$

$\ln \frac{58}{480} = 12k$

$k = \frac{\ln 58/480}{12}$ ← STO!

$\frac{1}{2} = e^{kt}$ ← use stored value

$\ln \frac{1}{2} = kt$

$\frac{\ln \frac{1}{2}}{k} = t$

$t \approx 3.9358$ minutes

f) If the half-life of a certain element is 2000 years. How long will it take the element to decrease to 75% of its original amount? Find k!

$\frac{1}{2} = e^{2000k}$

$\ln \frac{1}{2} = 2000k$

$k = \frac{\ln \frac{1}{2}}{2000}$ ← STO!

$\frac{3}{4} = e^{kt}$ ← use stored value! solve for t

$\ln \frac{3}{4} = kt$

$t = \frac{\ln \frac{3}{4}}{k}$

$t \approx 830$ yrs

#6) I CAN FIND THE NOMINAL OR EFFECTIVE INTEREST RATE.

(It would be easy to compare interest rates if all rates stated were nominal interest rates (also called simple interest rates) with the same compounding periods, but this is rarely the case. Because of this, we can calculate APY (the effective rate) as a method of comparing interest rates with different compounding periods.)

Calculate the following APYs (also called the effective rate).

a) Nominal interest rate of 7% compounded monthly

$\left(1 + \frac{.07}{12}\right)^{12}$

≈ 1.07229

effective rate: 7.229%

b) Simple interest rate of 6.5% compounded daily

↑ same as the nominal rate!

$\left(1 + \frac{.065}{365}\right)^{365}$

≈ 1.06715

effective rate: 6.715%

c) Nominal interest rate of 3.25% compounded annually

$\left(1 + \frac{.0325}{1}\right)^{1}$

Effective rate = nominal interest rate b/c it is compounded annually.

3.25%

- MAKE SURE THESE QUESTIONS SEEM EASY PEASY LEMON SQUEEZY!
- REVIEW QUIZ 5.1–5.3 AND THE REVIEW WORKSHEET FOR THE QUIZ.
- REVIEW YOUR HOMEWORK AND YOUR NOTES
- REVIEW UNTIL YOUR BRAIN HURTS!