5.3 – SOLVING QUADRATIC EQUATIONS

OBJECTIVES:
1) Solve a quadratic by factoring.
2) Solve a quadratic using square roots.
3) Solve a quadratic with quadratic formula.
4) Determine the number of solutions using the discriminant.

> **There are three different methods for algebraically solving a quadratic equation. Try to solve by a quadratic by using the MOST appropriate and efficient method.**

METHOD #1: SOLVE USING SQUARE ROOTS

This method is the quickest method, and the one you are most likely going to be comfortable with. The problem lies in the fact we can't always use it. You can't use it if there is a LINEAR TERM.

1) $3 - 9x^2 = -13$

2) $4(x+2)^2 + 7 = -9$

3) $x^2 - 4x = 5$

can't use square roots!

Factor!

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$\boxed{x = 5, -1}$

4) $\frac{1}{2}x^2 = 24$

$x^2 = 48$

$\boxed{x = \pm 4\sqrt{3}}$

5) $(x+5)^2 = 20$

6) $3(3x-2)^2 = 36$

$(3x-2)^2 = 12$

$3x - 2 = \pm\sqrt{12}$

$3x - 2 = \pm 2\sqrt{3}$

$3x = 2 \pm 2\sqrt{3}$

$\boxed{x = \dfrac{2 \pm 2\sqrt{3}}{3}}$

METHOD #2: FACTORING AND THE ZPP (ZERO PRODUCT PROPERTY)

Trinomial with lead coefficient of 1: $x^2 + bx + c = 0$	Trinomial with lead coefficient NOT 1: $ax^2 + bx + c = 0$	Difference of squares $a^2x^2 - b^2 = 0$
$-x^2 - 2x + 8 = 0$	$12x^2 - 10x = 8$	$x^2 - 16 = 0$

Ex 1) Solve: $3x^2 - 4x = 0$

Ex 2) Solve: $2x^2 - 9x = -12$

METHOD #3: QUADRATIC FORMULA

This method is YOUR LAST RESORT. It is tedious, and it's easy to make errors in computation, so when you can avoid using it, AVOID it. However, this method ALWAYS works, no matter what the original quadratic looks like.

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QUADRATIC FORMULA (MEMORIZE ME!):

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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Your quadratic must be in standard form:
$$ax^2 + bx + c = 0$$

DISCRIMINANT: $\sqrt{b^2 - 4ac}$

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1) $x^2 - 8x - 33 = 0$

2) $2x^2 + 9x = x^2 + 15x - 2$

$$x^2 - 6x + 2 = 0$$

$$x = \frac{6 \pm \sqrt{36 - 4(1)(2)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{28}}{2}$$

$$x = \frac{6 \pm 2\sqrt{7}}{2} = \boxed{3 \pm \sqrt{7}}$$

3) $2x^2 + 12x + 8 = 4x$

$$2x^2 + 8x + 8 = 0 \quad \text{Factor out 2!}$$

$$2(x^2 + 4x + 4) = 0 \quad (x+2)^2 \text{ but we'll still use}$$
$$\text{the quad. formula}$$

$$x = \frac{-4 \pm \sqrt{16 - 4(1)(4)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{0}}{2}$$

$$= \frac{-4}{2} \quad \boxed{x = -2}$$

4) $x^2 - 6x = -13$

$$x^2 - 6x + 13 = 0$$

$$x = \frac{6 \pm \sqrt{36 - 4(1)(13)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{-16}}{2} \quad \text{not real!}$$

$$\boxed{\text{no real solutions}}$$

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DISCRIMINANT: $\sqrt{b^2 - 4ac}$ — The discriminant can tell you how many solutions a quadratic has and gives you an idea about the graph of the function.

If it is **POSITIVE**:

If it is **NEGATIVE**:

If it is **ZERO**:

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