Ch 4
Day 10
Notes

ROW REDUCED ECHELON FORM

OBJECTIVES:

1) Solve a system of 2, 3 and 4 variable systems in row reduced echelon form.
2) Apply principles of multi-variable systems to real life scenarios.

Solve the system algebraically.

1) $2\left(\begin{cases} 3x - 2y = 6 \end{cases}\right)$
$-6x + 4y = 8$

$6x - 4y = 12$

$0 = 20$

False!

$\boxed{\text{No solution, lines are parallel}}$

Solve the system using inverse matrices.

2) $\begin{cases} 3x - 2y = 6 \\ -6x + 4y = 8 \end{cases}$

let $A = \begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = [A]^{-1}[B]$

ERROR!

ROW REDUCED ECHELON FORM

Directions: Solve using rref[ ]. The only way you want to do this is by using your calculator:

**Calculator Keystrokes*: Press $\boxed{2^{nd}}$ $\boxed{x^{-1}}$. Scroll over to MATH then scroll down to $\boxed{\text{B:rref(}}$

3) $\begin{cases} 3x - 2y = 6 \\ -6x + 4y = 8 \end{cases}$

let $A = \begin{bmatrix} 3 & -2 & 6 \\ -6 & 4 & 8 \end{bmatrix}$

$\text{rref}A = \begin{bmatrix} 1 & -2/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

No solution. Lines are parallel!

Solve the following systems using row reduced echelon form.

4) $\begin{cases} 11x - 5y = -38 \\ 9x + 2y = -25 \end{cases}$

$A = \begin{bmatrix} 11 & -5 & -38 \\ 9 & 2 & -25 \end{bmatrix}$

identity

$\text{rref}A = \begin{array}{c} x \quad y \quad \\ \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$

$\boxed{(-3, 1)}$

5) $\begin{cases} 8x - 12y = 24 \\ 6x - 9y = 18 \end{cases}$

$A = \begin{bmatrix} 8 & -12 & 24 \\ 6 & -9 & 18 \end{bmatrix}$

$\text{rref}A = \begin{bmatrix} 1 & -3/2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

$\boxed{\text{All points on the line } x - \frac{3}{2}y = 3}$

Interpreting Solutions:

6) $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -4 \end{bmatrix}$

$x = 3$
$y = -4$

$\boxed{(3, -4)}$

7) $\begin{bmatrix} 1 & -2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

$x - 2y = 3$

$\boxed{\text{Infinite sol. on } x - 2y = 3}$

8) $\begin{bmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$x - 2y = 0$

$0 = 1$ False!

$\boxed{\text{No solution, parallel lines!}}$

9) $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

$x = 2 \quad y = 3$
$z = 4$

$\boxed{(2, 3, 4)}$

10.) The number of cents per mile it costs to drive a car depends on how fast you drive it. At low speeds the cost is high because the engine operates inefficiently, while at high speeds the cost is high because the engine must overcome high wind resistance. At moderate speeds the cost reaches a minimum. Assume, therefore, that the cost varies **quadratically** with the speed of the car:

$$\text{cost} = a(\text{speed})^2 + b(\text{speed}) + c$$

a.) Suppose that at speed of 10, 20, and 30 mph the cost is 28, 21, and 16 cents per mile. What are the three ordered pairs?

$x = \text{speed}$
$y = \text{cost}$

$(10, 28) \quad (20, 21) \quad (30, 16)$

b.) Determine the three equations of your system.

$y = ax^2 + bx + c$

$28 = a \cdot 10^2 + b \cdot 10 + c$

$21 = a \cdot 20^2 + b \cdot 20 + c$

$16 = a \cdot 30^2 + b \cdot 30 + c$

$100a + 10b + c = 28$

$400a + 20b + c = 21$

$900a + 30b + c = 16$

c.) Solve the system for the coefficients a, b, and c. Show Work!

$$\text{let } A = \begin{bmatrix} 100 & 10 & 1 & 28 \\ 400 & 20 & 1 & 21 \\ 900 & 30 & 1 & 16 \end{bmatrix}$$

$a = .01$
$b = -1$
$c = 37$

$$\text{rref } A = \begin{bmatrix} 1 & 0 & 0 & .01 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 37 \end{bmatrix}$$

d.) Write down the final equation relating the cost to the speed.

$y = ax^2 + bx + c$

$\boxed{y = .01x^2 - x + 37}$

e.) How much would you spend to drive 65 mph?

$x = 65$

$y = .01(65)^2 - 65 + 37$

$y = 42.25 - 65 + 37$

$y = 14.25$

$\$.1425$
or
14.25¢ per mile