480/81
Day 10
Notes

8.6 Notes – Functions with More Than One Independent Variable

Objective:

- Given a real world situation involving more than one variable, find general and particular equations relating the variables, and use these as mathematical models.

Examples:

1. Y varies directly with the square of x, directly with z, and inversely with the square of w.
   a. Write the general equation.

   $y = \frac{kx^2z}{w^2}$

   b. Let x = 2, w = 5, z = 5, y = 12 and find k.

   $12 = \frac{k \cdot (2)^2 \cdot 5}{5^2}$

   $12 = \frac{4k}{5}$

   $\boxed{k=15}$ → $\boxed{y = \frac{15x^2z}{w^2}}$

2. Write an equation of variation where y varies jointly as x and z and inversely as the square of w.
   a. Write the general equation.

   $y = \frac{kxz}{w^2}$

   b. Let y= 27 when x = 15, z = 45 and w = 5 to find k.

   $27 = \frac{k \cdot 15 \cdot 45}{(5)^2}$

   $\boxed{k=1}$

   $\boxed{y = \frac{xz}{w^2}}$

Word Problems:

3. When a railroad track warms up, it expands. The amount by which it expands is directly proportional to the product of the length of the track section and the number of degrees by which it warms up. When tracks are installed, small gaps are left between the sections to allow for this expansion (see sketch). The width of the gap equals the amount by which the track is expected to expand. Suppose that you find that test rail section two meters long expands by 0.028 centimeter when it is warmed by 25°C.
   a. Write the particular equation expressing the width of the gap needed in terms of the rail length and the expected temperature rise.

   $G = .028\text{cm}$ $\ell = 2\text{m}$ $T = 25°C$

   G= width of gap & amount track expands

   ℓ= length of track

   T= amount temperature rises

   General Equation:
   $\boxed{G = k\ell T}$

   Particular:

   $G = k\ell T$

   $.028 = k(2)(25)$

   $k = \frac{.028}{50}$

   $k = 5.6 \times 10^{-4}$

   Particular Eq:
   $\boxed{G = 5.6 \times 10^{-4} \ell T}$

b. How wide a gap is need for

i. A 30-meter rail installed at 20°C in the tropics, where the rail temperatures can get as high as 60°C? $l = 30 \quad T = 60° - 20° = 40°$

$G = (5.6 \times 10^{-4})(30)(40)$

$\boxed{G = .672 \text{ cm}}$

ii. A 20-meter rail installed in the Antarctic at a temperature of -40°C, where the rail temperature could rise to as high as 32°C? $l = 20 \quad T = 32° - -40° = 72°$

$G = (5.6 \times 10^{-4})(20)(72)$

$\boxed{G = .8064 \text{ cm}}$

c. If a single rail with no gaps were laid in a straight line from San Francisco to New York (approximately 4100 km), by how much would it expand when the temperature goes from 10°C to 30°C?

$l = 4100 \text{ km} = 4{,}100{,}000 \text{ m} \quad T = 20°$

$G = (5.6 \times 10^{-4})(4{,}100{,}000)(20)$

$G = 45920 \text{ cm} \Rightarrow \boxed{459.2 \text{ m}}$

4. The amount of weight your thigh-bone will support is directly proportional to its cross sectional area. This area varies directly with the square of the diameter of the bone.

a. Write the general equation expressing weight in terms of area, and another general equation expressing the cross sectional area in terms of diameter.

W = amount of weight thigh-bone supports   D = diameter of bone

C = cross-sectional area

$\boxed{W = k_1 C}$ $\boxed{C = k_2 D^2}$

b. By composing the two functions in part a, derive an equation expressing the weight your thigh-bone will support in term of its diameter.

$W = k_1 k_2 D^2$

c. Tell in words how the weight varies with the diameter.

The weight varies directly w/ the square of the diameter.