480/81
Days 10
Notes

(PART 2) 7.6 NOTES – FACTOR THEOREM

OBJECTIVES:

1) Identify the possible rational zeros of a polynomial function.
2) Find all the roots of a polynomial function using your calculator!

We will still be finding the factors of our polynomial, but now, we'll have a little help from the calculator.

RATIONAL ROOT THEOREM:

$(ax-b)$ is a factor of $f(x)$

if and only if $f\left(\frac{b}{a}\right)=0$.

1) Factor: $f(x)=2x^3-x^2-x-3$

a) List all of the possible rational zeros.
b) Factor the polynomial.

a) $\pm\frac{p}{q}=\frac{\pm 1,3}{\pm 1,2}=\boxed{\pm 1,\ \frac{1}{2},\ 3,\ \frac{3}{2}}$

b) In calc: $f(1.5)=0$ $(1.5,0)$
$f(3/2)=0$

$\left(x-\frac{3}{2}\right)$ is a factor $\Rightarrow$ $(2x-3)$ is a factor

$\frac{3}{2}\ \Big|\ 1\ \ -\frac{1}{2}\ \ -\frac{1}{2}\ \ -\frac{3}{2}$
$\qquad\quad\ \ \frac{3}{2}\ \ \ \frac{3}{2}\ \ \ \frac{3}{2}$
$\qquad\ 1\ \ \ 1\ \ \ 1\ \ \ 0$

$\boxed{(2x-3)(x^2+x+1)}$

x^2+x+1 is prime

2) Factor: $f(x)=10x^4-3x^3-29x^2+5x+12$

a) List all of the possible rational zeros.
b) Factor the polynomial.

a) $\pm\frac{p}{q}=\frac{\pm 1,2,3,4,6,12}{\pm 1,2,5,10}=\boxed{\pm 1,\ \frac{1}{2},\ \frac{1}{5},\ \frac{1}{10},\ 2,\ \frac{2}{5},\ 3,\ \frac{3}{2},\ \frac{3}{5},\ \frac{3}{10},\ 4,\ \frac{4}{5},\ 6,\ \frac{6}{5},\ 12,\ \frac{12}{5}}$

b) $f(.8)=0 \Rightarrow f(4/5)=0$ so $(x-4/5)$ is a factor $(5x-4)$

$\frac{4}{5}\ \Big|\ 2\ \ -\frac{3}{5}\ \ -\frac{29}{5}\ \ 1\ \ \frac{12}{5}$
$\qquad\quad\ \ \frac{8}{5}\ \ \ \frac{4}{5}\ \ -4\ \ -\frac{12}{5}$
$\qquad\ 2\ \ \ 1\ \ -5\ \ -3\ \ 0$

$2x^3+x^2-5x-3$

$f(-1.5)=0$ $f(-3/2)=0$ $\left(x+\frac{3}{2}\right)\Rightarrow(2x+3)$ is a factor

$-\frac{3}{2}\ \Big|\ 1\ \ \frac{1}{2}\ \ -\frac{5}{2}\ \ -\frac{3}{2}$
$\qquad\quad\ \ -\frac{3}{2}\ \ \ \frac{3}{2}\ \ \ \frac{3}{2}$
$\qquad\ 1\ \ -1\ \ -1\ \ 0$

x^2-x-1 ↑ prime

$\boxed{(2x+3)(5x-4)(x^2-x-1)}$

3) Factor: $f(x) = 12x^3 + 16x^2 - 5x - 3$

a) List all of the possible rational zeros.

b) Factor the polynomial.

a) $\pm\frac{p}{q} = \frac{\pm 1, 3}{\pm 1, 2, 3, 4, 6, 12} = \boxed{\pm 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{12}, 3, \frac{3}{2}, \frac{3}{4}}$

b) $f(-.33) = 0$ $f(-\frac{1}{3}) = 0$ $\left(x + \frac{1}{3}\right)$ is a factor, so $(3x+1)$ is a factor.

$$\begin{array}{r|cccc} -\frac{1}{3} & 4 & \frac{16}{3} & -\frac{5}{3} & -1 \\ & & -\frac{4}{3} & -\frac{4}{3} & 1 \\ \hline & 4 & 4 & -3 & 0 \end{array}$$

$4x^2 + 4x - 3$

$(2x + 3)(2x - 1)$

Factors:

$\boxed{(3x+1)(2x+3)(2x-1)}$