480/81
Day 11
Notes

KEY

6.11 NOTES – CHANGE OF BASE AND LOG EQUATIONS

OBJECTIVES:

1) Use common logs to solve equations.
2) Apply the change of base formula.

SOLVE LOGARITHM EQUATIONS BY APPLYING LOG PROPERTIES:

1) $2\log_3 x = \log_3 9$

$2\log_3 x = 2$ OR $\log_3 x^2 = \log_3 9$

$\log_3 x = 1$ $\quad x^2 = 9$

$3^1 = x$ $\quad \boxed{x = 3}$ $\quad x = -3$

$x = 3$

2) $\log_7 x = \frac{2}{3}\log_7 8$

$\log_7 x = \log_7 (2^3)^{2/3}$

$\log_7 x = \log_7 2^2$

$\boxed{x = 4}$

3) $2\log_6 x + \log_6 4 = \log_6 64$

$\log_6 (x^2 \cdot 4) = \log_6 64$

$4x^2 = 64$

$x^2 = 16$

$\boxed{x = 4}$ $\quad x = -4$

4) $\log_8 48 - \log_8 y = \log_8 4$

$\log_8 \frac{48}{y} = \log_8 4$

$\frac{48}{y} = 4$

$\boxed{y = 12}$

5) $\log_3 x = 5\log_3 2 - \log_3 8$

$\log_3 x = \log_3 \frac{2^5}{8}$

$x = \frac{32}{8}$

$\boxed{x = 4}$

6) $\log_2(3u + 14) - \log_2 5 = \log_2 2u$

$\log_2 \left(\frac{3u+14}{5}\right) = \log_2 2u$

$\frac{3u+14}{5} = 2u$

$3u + 14 = 10u$

$14 = 7u$

$\boxed{u = 2}$

7) $\ln(x+3) + \ln x = \ln 4$

$\ln((x+3)\cdot x) = \ln 4$

$x^2 + 3x = 4$

$x^2 + 3x - 4 = 0$

$(x+4)(x-1) = 0$

$x = -4$ $\quad \boxed{x = 1}$

8) $\ln 4 + \ln x = \ln e^2$

$\ln 4x = \ln e^2$

$4x = e^2$

$\boxed{x = \frac{e^2}{4}}$

9) $3\log_5(x^2 + 9) - 6 = \log_5 1$

$3\log_5(x^2+9) - 6 = 0$

$3\log_5(x^2+9) = 6$

$\log_5(x^2+9) = 2$

$5^2 = x^2 + 9$

$16 = x^2$

$\boxed{x = \pm 4}$

CHANGE OF BASE FORMULA:

Why would we ever want to change the base of our logarithm? Well, the reason is that we cannot evaluate a logarithm like $\log_5 7$ in our heads.

★ CHANGE OF BASE FORMULA: ★

$$\log_a c = \frac{\log c}{\log a}$$ (Base 10 can be put in our calculator!)

Examples:

1) $\log_4 25 =$

$\frac{\log 25}{\log 4}$

≈ 2.322

2) $\log_3 18 =$

$\frac{\log 18}{\log 3}$

≈ 2.631

3) $\log_6 \sqrt{5} =$

$\frac{\log \sqrt{5}}{\log 6}$

$\approx .449$

USE LOGS TO SOLVE EXPONENTIAL EQUATIONS:

State your answer as an exact answer and then approximate your answer to the nearest thousandths.

1) $3^x = 27$

$3^x = 3^3$

$x = 3$

2) $5^x = 120$

$\log_5 120 = x$

$\frac{\log 120}{\log 5} = x$

≈ 2.975

3) $e^x = 52$

$\ln 52 = x$

≈ 3.951

4) $4^{2x} = 27$

$\log_4 27 = 2x$

$\frac{\log_4 27}{2} = x$

≈ 1.189

5) $2^{x-4} = 82$

$\log_2 82 = x - 4$

$\log_2 82 + 4 = x$

≈ 10.358

6) $e^{2x-3} = 42$

$\ln 42 = 2x - 3$

$\frac{\ln 42 + 3}{2} = x$

≈ 3.369

7) $5^{x-3} = 72$

$\log_5 72 = x - 3$

$\log_5 72 + 3 = x$

≈ 5.657

8) $4 + e^{\frac{x}{3}} = 10$

$e^{\frac{x}{3}} = 6$

$\ln 6 = \frac{x}{3}$

$3 \ln 6 = x$

≈ 5.375