

4.4 – F(X) NOTATION

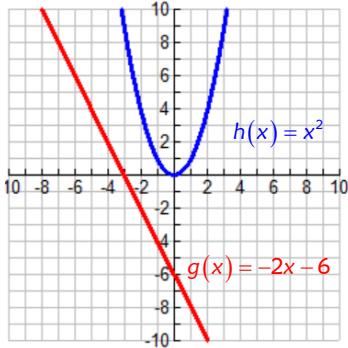
OBJECTIVES:

- Find the sum, difference, product, and quotient of functions.
- Find the value of a function for a given value of x .
- Find the composition of functions.

REMEMBER!
 $f(x)$ is not “f times x”!

$$f(x) = \underline{\hspace{2cm}}$$

EVALUATING FUNCTIONS WITH A GRAPH



Use the graphs above to compute the following:

$$1) h(2) = 4 \\ (2, 4)$$

$$2) g(2) = -10 \\ (2, -10)$$

$$3) \frac{g(0)}{h(2)} = \frac{-6}{4} = \boxed{\frac{-3}{2}}$$

$$4) h(g(-3))$$

$g(-3) = 0$ so find $h(0)$.

$$h(0) = 0$$

$$h(g(-3)) = 0$$

WITHOUT A GRAPH

Let $f(x) = -x^3 + 2x^2$ and $g(x) = -6x - 2$. Evaluate the following:

$$5) f(2)$$

$$f(2) = -(2)^3 + 2(2)^2$$

$$= -8 + 8$$

$$f(2) = \boxed{0}$$

$$6) g(-3)$$

$$g(-3) = -6(-3) - 2$$

$$= 18 - 2$$

$$= \boxed{16}$$

$$7) \frac{f(-1)}{g(1)} = \boxed{\frac{3}{-8}}$$

$$f(-1) = -(-1)^3 + 2(-1)^2$$

$$= 1 + 2 = 3$$

$$g(1) = -6(1) - 2 = -8$$

RULES:

- Sum: $(f+g)(x) = f(x) + g(x)$
- Difference: $(f-g)(x) = f(x) - g(x)$
- Product: $(fg)(x) = f(x) \cdot g(x)$
- Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

If $f(x) = 2x + 5$ and $g(x) = x - 1$ find:

$$8) (f+g)(x)$$

$$(f+g)(x) = f(x) + g(x) \\ = 2x + 5 + x - 1$$

$$(f+g)(x) = \boxed{3x + 4}$$

$$9) (f-g)(x)$$

$$(f-g)(x) = f(x) - g(x) \\ = 2x + 5 - (x - 1)$$

$$(f-g)(x) = \boxed{x + 6}$$

$$10) (fg)(x)$$

$$(fg)(x) = f(x) \cdot g(x) \\ = (2x + 5)(x - 1)$$

$$= 2x^2 - 2x + 5x - 5$$

$$(fg)(x) = \boxed{2x^2 + 3x - 5}$$

$$11) \left(\frac{f}{g}\right)(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Can't simplify!!

$$\left(\frac{f}{g}\right)(x) = \boxed{\frac{2x + 5}{x - 1}}$$

COMPOSITION: Use the output from one function as the input for another.
 "f of g of x" "g of f of x"

$$(f \circ g)(x) = f(g(x))$$

$$(g \circ f)(x) = g(f(x))$$

To simplify: 1) Plug x into g.
 2) Use the result from step 1 in f.

If $f(x) = 3x^2$ and $g(x) = 2x - 1$, find:

$$12) (f \circ g)(3) = f(g(3))$$

$$\begin{aligned} g(3) &= 2(3) - 1 \\ &= 5 \\ f(5) &= 3(5)^2 \\ &= 3(25) \\ &= 75 \\ \boxed{f(g(3))} &= 75 \end{aligned}$$

$$13) (g \circ f)(3) = g(f(3))$$

$$\begin{aligned} f(3) &= 3(3)^2 = 27 \\ g(27) &= 2(27) - 1 \\ &= 54 - 1 \\ &= 53 \\ \boxed{g(f(3))} &= 53 \end{aligned}$$

$$14) (f \circ f)(3) = f(f(3))$$

$$\begin{aligned} f(3) &= 3(3)^2 = 27 \\ f(27) &= 3(27)^2 \\ &= 3(\text{A big #}) \\ &\quad \begin{matrix} \cancel{\text{not so bad!}} \\ \text{Sorry!} \end{matrix} \\ &= 3(729) \\ &= 2187 \\ \boxed{f(f(3))} &= 2187 \end{aligned}$$

$$15) (f \circ g)(x) = f(g(x))$$

$$\begin{aligned} f(x) &= 3x^2 \\ f(g(x)) &= 3(2x - 1)^2 \\ &= 3(4x^2 - 4x + 1) \\ \boxed{f(g(x))} &= (2x^2 - 12x + 3) \end{aligned}$$

$$16) (g \circ f)(x)$$

$$\begin{aligned} g(x) &= 2x - 1 \\ g(f(x)) &= 2(3x^2) - 1 \\ \boxed{g(f(x))} &= 6x^2 - 1 \end{aligned}$$

If $f(x) = x - 1$ and $g(x) = x^2 - 3x + 1$, find:

$$17) (g \circ f)(x)$$

$$\begin{aligned} g(x) &= x^2 - 3x + 1 \\ g(f(x)) &= (x-1)^2 - 3(x-1) + 1 \\ &= x^2 - 2x + 1 - 3x + 3 + 1 \\ \boxed{g(f(x))} &= x^2 - 5x + 5 \end{aligned}$$

$$18) (f \circ g)(x)$$

$$\begin{aligned} f(x) &= x - 1 \\ f(g(x)) &= x^2 - 3x + 1 - 1 \\ \boxed{f(g(x))} &= x^2 - 3x \end{aligned}$$

YOU TRY!

- 1) Find $(f + g)(x)$ and $(f - g)(x)$, when $f(x) = 3x^3 - 2x^2 + 5x - 1$, $g(x) = x^2 + 7x - 1$. Simplify your answer.

$$(f+g)(x) = 3x^3 - 2x^2 + 5x - 1 + (x^2 + 7x - 1) \\ = \boxed{3x^3 - x^2 + 12x - 2}$$

$$(f-g)(x) = 3x^3 - 2x^2 + 5x - 1 - (x^2 + 7x - 1) \\ = 3x^3 - 2x^2 + 5x - 1 - x^2 - 7x + 1 \\ = \boxed{3x^3 - 3x^2 - 2x}$$

- 2) Find $(f \cdot g)(x)$, when $f(x) = -x + 2$, $g(x) = x + 1$. Simplify your answer.

$$(f \cdot g)(x) = (-x + 2)(x + 1) \\ = -x^2 - x + 2x + 2 \\ = \boxed{-x^2 + x + 2}$$

- 3) Find $\left(\frac{f}{g}\right)(x)$, when $f(x) = 6x^{\frac{7}{3}}$, $g(x) = 3x^{\frac{2}{3}}$. Simplify your answer. **HINT:** $\frac{12x^9}{6x^6} = 2x^3$

$$\left(\frac{f}{g}\right)(x) = \frac{6x^{\frac{7}{3}}}{3x^{\frac{2}{3}}} = \boxed{2x^{\frac{5}{3}}} \quad (\text{just subtract the exponents})$$

- 4) Let $f(x) = 2x^2 - 3$ and $g(x) = -6x + 2$

a. $f(g(x))$

$$f(x) = 2x^2 - 3$$

$$f(g(x)) = 2(-6x + 2)^2 - 3$$

$$= 2(36x^2 - 24x + 4) - 3$$

$$= 72x^2 - 48x + 8 - 3$$

$$= \boxed{72x^2 - 48x + 5}$$

b. $g(f(x))$

$$g(x) = -6x + 2$$

$$g(f(x)) = -6(2x^2 - 3) + 2$$

$$= -12x^2 + 18 + 2$$

$$= \boxed{-12x^2 + 20}$$

c. $f(f(x))$

$$f(x) = 2x^2 - 3$$

$$f(f(x)) = 2(2x^2 - 3)^2 - 3$$

$$= 2(4x^4 - 12x^2 + 9) - 3$$

$$= 8x^4 - 24x^2 + 18 - 3$$

$$= \boxed{8x^4 - 24x^2 + 15}$$

d. $g(f(1))$

$$f(1) = 2(1)^2 - 3$$

$$= -1$$

$$g(-1) = -6(-1) + 2$$

$$= 6 + 2$$

$$= \boxed{8}$$

$$g(f(1)) = \boxed{8}$$

Check with the key! You'll have a video quiz check on one of these types of problems tomorrow!