8.6 – COMPOSITE FUNCTIONS

OBJECTIVE:

- Given a real world situation involving more than one variable, find general and particular equations relating the variables, and use these as mathematical models.

"WHY MAMMALS ARE THE WAY THEY ARE" (#11 on p.450)

1. Some physical and behavior characteristics of mammals can be explained by comparing the way their surface area and mass are related. Assume that:

   m = mass, ℓ = length
   i. the mass of an animal is directly proportional to the cube of its length,
   s = skin area
   ii. the area of its skin is directly proportional to the square of its length,
   h = heat loss
   iii. the rate at which it loses heat to its surroundings is directly proportional to its skin area, and
   f = amount of food
   iv. the amount of food it must eat per day is directly proportional to the rate of heat loss.

   a. Write general equations for each of the four functions above.

   i. $m = k_1 \ell^3$

   ii. $s = k_2 \ell^2$ $\qquad \ell = \sqrt[3]{\frac{m}{k_1}}$

   iii. $h = k_3 s$

   iv. $f = k_4 h$

   b. By appropriate substitutions, derive a general equation expressing amount of food per day in terms of the animal's mass. Tell how the food consumption varies with mass.

   $f = k_4 h \rightarrow h = k_3 s$

   $f = k_4 k_3 s \rightarrow s = k_2 \ell^2$

   $f = k_4 k_3 k_2 \ell^2 \rightarrow \ell = \sqrt[3]{\frac{m}{k_1}}$

   $f = k_4 k_3 k_2 \left(\sqrt[3]{\frac{m}{k_1}}\right)^2$

   SIMPLIFY: $f = k_5 \sqrt[3]{m^2} = k_5 m^{2/3}$

   Food consumption varies directly w/ the 2/3 power of mass.

   c. The smallest mammal, the shrew, eats 3 times its mass each day. A shrew has a mass of about 2 grams (about that of a penny!). Find the proportionality constant for the equation in part b.

   $m = 2$ grams

   $F = 6$ grams

   $F = k_5 m^{2/3}$

   $6 = k_5 (2)^{2/3}$

   $k_5 = \frac{6}{2^{2/3}} \approx 3.78$

   $k_5 \approx 3.78$

   STO!

d. What amount of food would a 6000 kilogram elephant eat each day?

$m = 6{,}000 \text{ kg} = 6{,}000{,}000 \text{ grams}$

$F = 3.78(6{,}000{,}000)^{2/3}$

$F \approx 124805.03 \text{ grams}$

$\approx 124.8 \text{ kg}$

"WEATHER BALLOON PROBLEM"

2. Helium-filled balloons are sent up into the atmosphere carrying instruments that measure weather conditions. As the balloon ascends, assume that:

V = volume
T = temp.
P = pressure
R = radius
A = surf. area

   i. its volume varies directly with the absolute temperature and inversely with the pressure of the atmosphere,
   ii. the volume of the balloon is also directly proportional to the cube of its radius, and
   iii. its surface area is directly proportional to the square of its radius.

a. Write general equations for each of the three functions above.

   i. $V = \frac{k_1 T}{P}$

   ii. $V = k_2 R^3 \longrightarrow R = \sqrt[3]{\frac{V}{k_2}}$

   iii. $A = k_3 R^2$

b. By appropriate substitutions, derive an equation for the surface area in terms of volume.

$A(v)$

$A = k_3 \underline{R^2} \longrightarrow R = \sqrt[3]{\frac{V}{k_2}}$

$A = k_3\left(\sqrt[3]{\frac{V}{k_2}}\right)^2$ simplify: $A = k_4 V^{2/3}$

c. By appropriate substitutions, write an equation expressing the surface area in terms of the temperature and pressure.

$A = k_4 \underline{V}^{2/3} \longrightarrow V = \frac{k_1 T}{P}$

$A = k_4\left(\frac{k_1 T}{P}\right)^{2/3}$ SIMPLIFY: $\boxed{A = \frac{k_5 T^{2/3}}{P^{2/3}}}$