480/81
Day 13
Notes

6.12 INVERSE FUNCTIONS

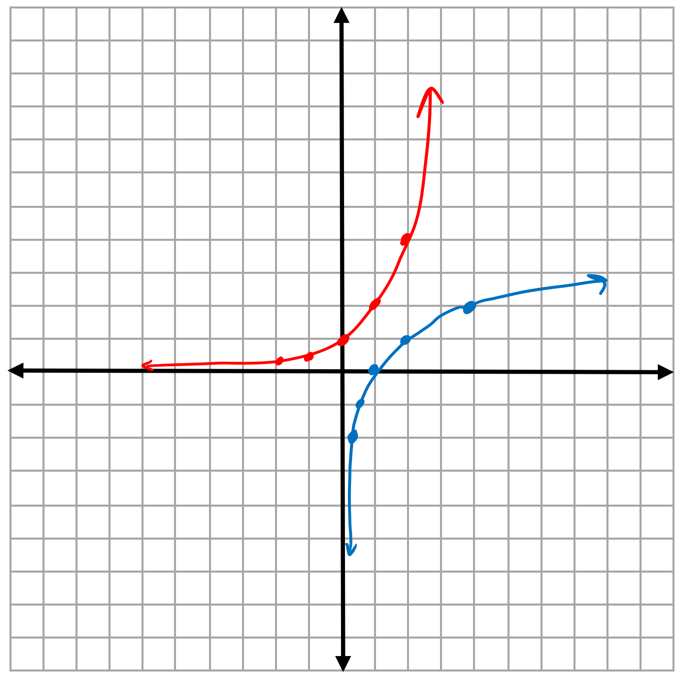
OBJECTIVES:

1) Use the definition of inverse functions to determine if a function is an inverse of another.
2) Find the inverse of a function.

WARM-UP REVIEW

1. Sketch the graph of $y = \log_2 x$. $2^y = x$

x	y
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2



x-int: $(1,0)$ y-int: DNE.

Domain: $x > 0$ Range: $\mathbb{R}$

Asymptote: $x = 0$

2. On the same grid, graph $y = 2^x$.

x	y
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

x-int: D.N.E. y-int: $(0,1)$

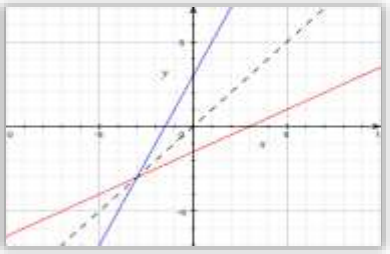
Domain: $\mathbb{R}$ Range: $y > 0$

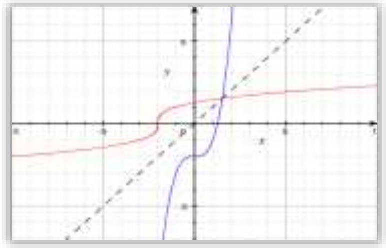
Asymptote: $y = 0$

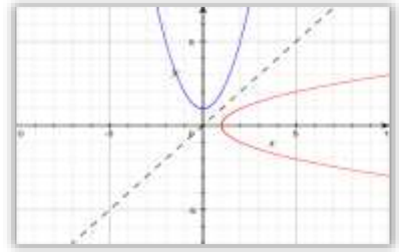
How are the two graphs related, similar, and different?

* Watch the video!

INVERSES:
A function must be 1 to 1 in order for it to have an inverse function.







$f^{-1}(x)$ is not a function!

$f(x)$ is <u>not</u> 1-1

GRAPHING INVERSE FUNCTIONS:

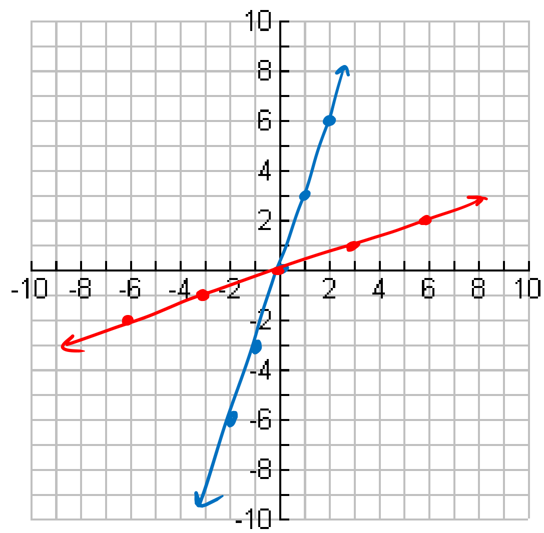
Graph the function and its inverse.

3) $f(x) = 3x$

$f(x) \longrightarrow f^{-1}(x)$

x	y
-2	-6
-1	-3
0	0
1	3
2	6

x	y
-6	-2
-3	-1
0	0
3	1
6	2

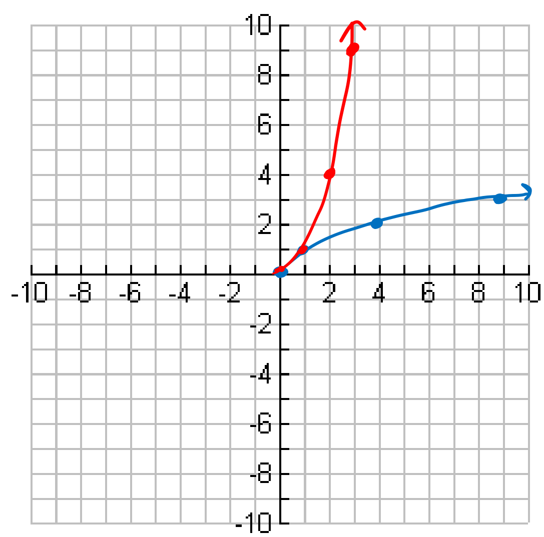


4) $f(x) = \sqrt{x}$

$f(x) \longrightarrow f^{-1}(x)$

x	y
0	0
1	1
4	2
9	3

x	y
0	0
1	1
2	4
3	9



DEFINITION OF INVERSE FUNCTIONS:

Two functions f and g are inverses of one another if and only if: $f(g(x)) = x$ and $g(f(x)) = x$

5) Determine if the following are inverses of one another.

a) $f(x) = \frac{1}{2}x - 4$ and $g(x) = 2x + 8$

$f(g(x)) = x$

$f(g(x)) = \frac{1}{2}(2x+8) - 4$

$f(g(x)) = x + 4 - 4 = x$

$f(g(x)) = x$ ✓

$g(f(x)) = x$

$g(f(x)) = 2(\frac{1}{2}x - 4) + 8$

$= x - 8 + 8$

$= x$

$g(f(x)) = x$ ✓

b) $f(x) = 3x - 1$ and $g(x) = \frac{x}{3} + 1$

$f(g(x)) = f(\frac{x}{3} + 1)$

$= 3(\frac{x}{3} + 1) - 1$

$= x + 3 - 1$

$= x + 2$

$f(g(x)) \neq x$ so they are <u>not</u> inverses.

FINDING THE INVERSE OF A FUNCTION ALGEBRAICALLY:

1. Use x and y notation.
2. Exchange x and y.
3. Solve for y.
4. Replace $f^{-1}(x)$ notation

1. $f(x) = 2x + 7$

1) $y = 2x + 7$

2) $x = 2y + 7$

3) $x - 7 = 2y$

$\frac{x-7}{2} = y$

4) $\boxed{f^{-1}(x) = \frac{x-7}{2}}$

2. $g(x) = \frac{3}{2}x - 6$

$y = \frac{3}{2}x - 6$

$x = \frac{3}{2}y - 6$

$x + 6 = \frac{3}{2}y$

$\frac{2}{3}(x+6) = y$

$y = \frac{2}{3}x + 4$

$\boxed{f^{-1}(x) = \frac{2}{3}x + 4}$

3. $f(x) = 5^x$

$y = 5^x$

$x = 5^y$

$\log_5 x = y$

$\boxed{f^{-1}(x) = \log_5 x}$

4. $f^{-1}(x) = 6 + \log_3 x$

$y = 6 + \log_3 x$

$x = 6 + \log_3 y$

$x - 6 = \log_3 y$

$3^{x-6} = y$

$\boxed{f^{-1}(x) = 3^{x-6}}$