

6.14 NOTES – EXPONENTIAL MODELS

OBJECTIVES:

- 1) Given a real-world situation relating two variables, use an exponential, linear, or quadratic function as a mathematical model.

“POPULATION PROBLEM”

Assume that the population of the United States is increasing exponentially with time. The 1970 census showed that the population was about 203 million. The 1980 census showed that the population had grown to about 225 million.

- 1) Find the particular equation expressing population in terms of the number of years that have elapsed since 1970. Define variables and write ordered pairs, then find the exponential equation in the form $y = a \cdot b^x$.

$$t = 0 \text{ in } 1970 \quad (0, 203) \quad (10, 225)$$

$$y = 203(b)^x$$

$$225 = 203(b)^{10}$$

$$\frac{225}{203} = b^{10}$$

$$b = \left(\frac{225}{203}\right)^{\frac{1}{10}}$$

$$y = 203 \left(\frac{225}{203}\right)^{\frac{1}{10}x}$$

STO B!

- 2) Use your equation to predict the population this year.

$$y = 203b^x$$

store this value!

- 3) Predict the year in which the population will reach 400 million.

$$400 = 203b^x$$

$$\frac{400}{203} = b^x$$

$$\log_b \frac{400}{203} = x$$

use change of base!

- 4) According to your model, what was the population when the Declaration of Independence was signed?

$$x = -174$$

$$y = 203b^x$$

$$y = 203b^{-174} \approx 27.578$$

Model predicts
about 27.6 million people.

- 5) The actual population when the Declaration of Independence was signed was only 4 million. Explain why your model predicted a much larger population.

Extrapolation vs. Interpolation!