

6.9 NOTES – LOGS WITH OTHER BASES

KEY

OBJECTIVES:

- 1) Rewrite an exponential equation as a logarithm.
- 2) Evaluate a log.
- 3) Solve a logarithmic equation.

LOGARITHM: A logarithm is just an EXPONENT !

$$y = \log_b x \quad \text{if and only if} \quad b^y = x, \text{ where } x > 0, b > 0, b \neq 1$$

y is the logarithm

b is the base

x is the argument

2^5 the exponent is called a base 2 logarithm.
 $5 = \log_2 32$ 5 is "log to the base 2 of 32"

Fill in the table below.

EXPONENTIAL FORM	LOGARITHMIC FORM
$3^4 = 81$	$\log_3 81 = 4$
$6^3 = 216$	$\log_6 216 = 3$
$3^{-4} = \frac{1}{81}$	$\log_3 \frac{1}{81} = -4$
$10^{-5} = 0.0001$	$\log 0.0001 = -5$
$2^{-5} = \frac{1}{32}$	$\log_2 \frac{1}{32} = -5$
$25^{\frac{1}{2}} = 5$	$\log_{25} 5 = \frac{1}{2}$
$32^{\frac{3}{5}} = 8$	$\log_{32} 8 = \frac{3}{5}$
$\left(\frac{1}{4}\right)^3 = \frac{1}{64}$	$\log_{\frac{1}{4}} \frac{1}{64} = 3$

$\log x = \log_{10} x$

EVALUATING LOGS

1) If $3^4 = 81$, then $\log_3 81 = 4$

2) If $9^0 = 1$, then $\log_9 1 = 0$

3) $\log_2 64$

$2^? = 64$
6

4) $\log_4 \frac{1}{16}$

$4^? = \frac{1}{16}$
-2

5) $\log_3 243$

$3^? = 243$
5

6) $\log_{25} 5$

$25^? = 5$
 $(5^2)^? = 5$
1/2

7) $\log_7 \frac{1}{49}$

$7^? = \frac{1}{49}$ $7^? = 7^{-2}$
-2

8) $\log_8 4$

$8^? = 4$
 $(2^3)^? = 2^2$
2/3

9) $\log_6 6^8$

$6^? = 6^8$
8

10) $3^{\log_3 11}$

$\log_3 ? = \log_3 11$
11

SOLVING LOGARITHMIC EQUATIONS

A logarithmic equation can ask you one of several things:

- 1) to find the argument
- 2) to find the logarithm/exponent
- 3) to find the base

FINDING THE ARGUMENT:

11) $\log_3 x = -4$

$$3^{-4} = x$$

$$x = \frac{1}{81}$$

12) $\log_{\sqrt{2}} x = 6$

$$(2^{\frac{1}{2}})^6 = x$$

$$2^3 = x$$

$$x = 8$$

13) $\log_{\frac{1}{2}} x = -4$

$$\left(\frac{1}{2}\right)^{-4} = x$$

$$(2^{-1})^{-4} = x$$

$$2^4 = x$$

$$x = 16$$

14) $\log_8 x = \frac{4}{3}$

$$8^{\frac{4}{3}} = x$$

$$(2^3)^{\frac{4}{3}} = x$$

$$2^4 = x$$

$$x = 16$$

FINDING THE LOGARITHM/EXPONENT:

15) $\log_2 8 = x$

$$2^x = 8$$

$$2^x = 2^3$$

$$x = 3$$

16) $\log_{27} 81 = x$

$$27^x = 81$$

$$(3^3)^x = 3^4$$

$$3x = 4$$

$$x = \frac{4}{3}$$

17) $\log_{\frac{1}{4}} 16 = x$

$$\left(\frac{1}{4}\right)^x = 16$$

$$(4^{-1})^x = 4^2$$

$$-x = 2$$

$$x = -2$$

18) $\log_{-\frac{1}{4}} 64 = x$

NO SOLUTION!

can't have a negative base!

FINDING THE BASE:

19) $\log_x 4 = \frac{2}{3}$

$$x^{\frac{2}{3}} = 4$$

$$\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = (2^2)^{\frac{3}{2}}$$

$$x = 2^3$$

$$x = 8$$

20) $\log_x \frac{1}{16} = 2$

$$x^2 = \frac{1}{16}$$

$$x^2 = 4^{-2}$$

$$x = \pm \frac{1}{4}$$

$$x = \frac{1}{4}$$

~~$$x = -\frac{1}{4}$$~~

can't have a negative base

21) $\log_x \frac{1}{4} = -2$

$$x^{-2} = \frac{1}{4}$$

$$(x^{-2})^{-\frac{1}{2}} = (2^{-2})^{-\frac{1}{2}}$$

$$x = 2^1$$

$$x = 2$$

22) $\log_x 4 = 0$

$$x^0 = 4$$

impossible

NO SOLUTION!

ALWAYS CHECK YOUR SOLUTIONS!