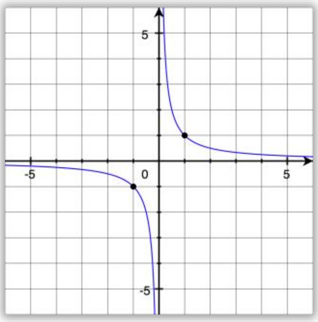
4.7
Notes
Day 1

RATIONAL FUNCTIONS

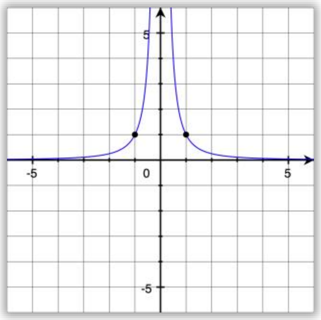
Objectives: 1) Graph rational functions.

MEMORIZE: Rational power functions.
Know domain, range, vertical asymptote(s), horizontal asymptote

$f(x) = \frac{1}{x}$

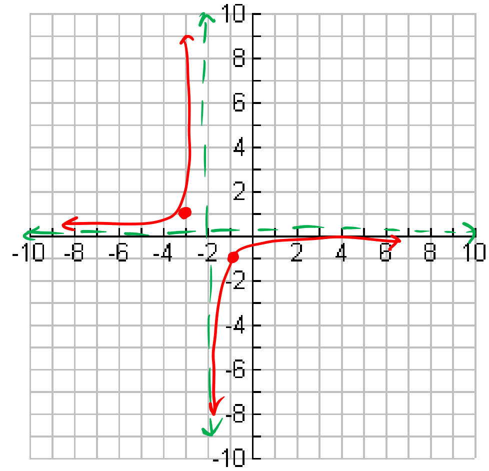


$f(x) = \frac{1}{x^2}$



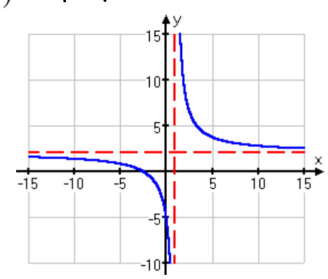
1) How would you graph $f(x) = \frac{-3}{x+2}$?

F X
S·3
L2



RATIONAL FUNCTION: $r(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomial functions.

- A rational function can have both vertical AND horizontal asymptotes.
- A rational function has branches – one more branch than the number of vertical asymptotes.



1 vert. asymp.

2 branches

FINDING VERTICAL ASYMPTOTES:

To find the vertical asymptotes, set the denominator equal to 0 and solve for x. There may be more than one!

1) $y = \frac{x-3}{x^2-9}$ $\frac{\cancel{(x-3)}}{(x+3)\cancel{(x-3)}}$

VA: $x = -3$ (Hole at $x = 3$)

2 branches

2) $y = \frac{x^2-4}{x^2+4x-5}$

Factor: $\frac{(x+2)(x-2)}{(x+5)(x-1)}$

VA: $x = -5$
$x = 1$

3 branches

3) $y = \frac{x^2}{x+2}$

VA: $x = -2$

2 branches

4) $y = \frac{1}{x^2+2}$

no vertical asymptotes!
Denom can never $= 0$

1 branch

HOW MANY BRANCHES WOULD EACH GRAPH HAVE?

AN EXAMPLE OF A GRAPH OF RATIONAL FUNCTION: $f(x)=\dfrac{x^2-3x+2}{x^2-6x+8}$

1) Factor the numerator and denom.

2) Find any holes.
Hole - occurs when factors cancel

3) Find the x-intercept(s). Let y = 0.

4) Find the y-intercept. Let x = 0.

5) Find all vertical asymptotes.
Set denominator = 0.

6) Find the horizontal asymptote.

7) Sketch the branches of the curve.

1) $f(x)=\dfrac{x^2-3x+2}{x^2-6x+8}=\dfrac{(x-2)(x-1)}{(x-2)(x-4)}$

2) $f(x)=\dfrac{\cancel{(x-2)}(x-1)}{\cancel{(x-2)}(x-4)}$ Hole: x = 2. Find f(2): $f(2)=\dfrac{(2-1)}{(2-4)}=\dfrac{1}{-2}$

HOLE: $\left(2,\dfrac{1}{-2}\right)$

3) x – int: $0=\dfrac{(x-1)}{(x-4)}$

$0=(x-1)$

$x=1$

4) y – int: $y=\dfrac{(0-1)}{(0-4)}$

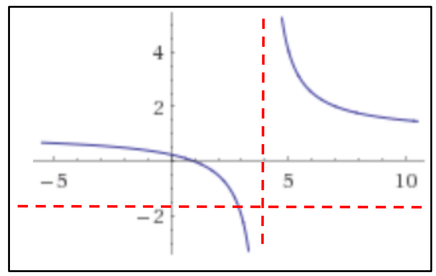
$y=\dfrac{1}{4}$

5) VA: $0=(x-4)$ (denominator = 0)

VA: $x=4$

6) HA: Degree is same! $y=\dfrac{1x^2}{1x^2}=1$ HA: $y=1$

7)



FINDING HORIZONTAL ASYMPTOTE

To find the horizontal asymptote, compare the degree of the numerator(n) to the degree of the denominator(d).

There are three different cases: (The maximum number of horizontal asymptotes is one.)

CASE 1: $n>d$

1) $f(x)=\dfrac{-3x^3+x-1}{x-2}$

No horizontal asymptote!

CASE 2: $n=d$

2) $f(x)=\dfrac{4x^2+1}{x^2-7}$

$y=\dfrac{\text{lead coefficient of numerator}}{\text{lead coefficient of denominator}}$

CASE 3: $n<d$

3) $f(x)=\dfrac{2x+2}{x^2-3}$

y=0

SOME RHYMES TO HELP YOU REMEMBER: (They are terrible, I know.)

1) "IF YOU BIG ON TOP, JUST STOP!" (CUZ THEY AIN'T NO HORIZONTAL ASYMPTOTE)

2) Got nothin' for this one. Ain't nobody got time for that.

3) "IF THE BOTTOM IS HEAVY, THE LINE IS STEADY (AT Y = 0)".

Graph the following rational functions:

1) $f(x)=\dfrac{1}{x+3}$

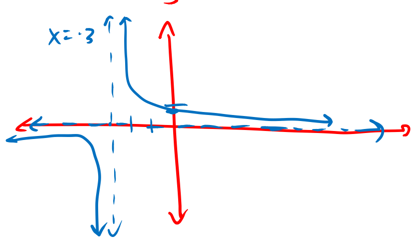
No holes

VA: $x=-3$

HA: $y=0$

x int: $0=1$ no x int.

y int: $\frac{1}{3}=y$ $(0,\frac{1}{3})$



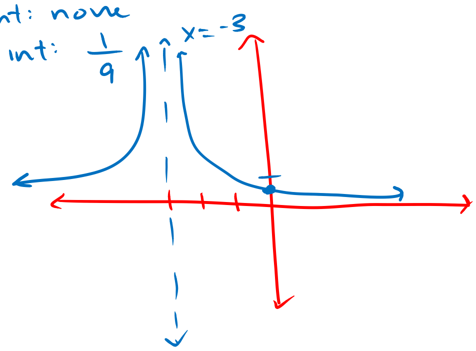
2) $f(x)=\dfrac{1}{(x+3)^2}$ **NOT** $f(x)=\dfrac{1}{x^2+3}$

VA: $x=-3$

HA: $y=0$

x int: none

y int: $\frac{1}{9}$



1) Factor the numerator and denom.

2) Find any holes.
Hole - occurs when factors cancel

3) Find the x and y-intercepts.

5) Find all vertical asymptotes.
Set denominator = 0.

6) Find the horizontal asymptote.
Analyze long run behavior.

7) Sketch the branches of the curve.

3) $f(x)=\dfrac{x+1}{(x-3)^2}$

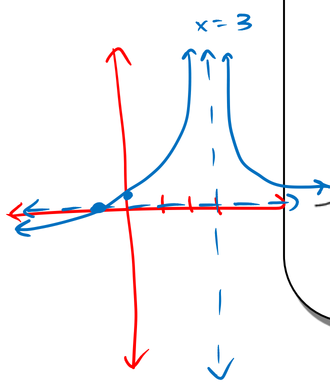
no holes

VA: $x=3$

HA: $y=0$

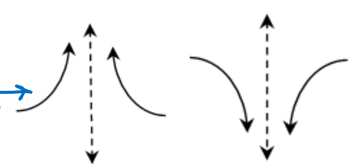
x int: -1 $(-1,0)$

y int: $(0,\frac{1}{9})$



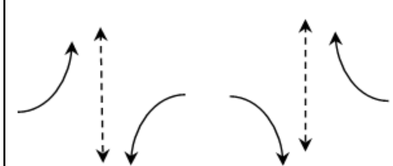
EVEN MULTIPLICITIES

$\dfrac{1}{(x-1)^{even}}$



ODD MULTIPLICITIES

$\dfrac{1}{(x-1)^{odd}}$



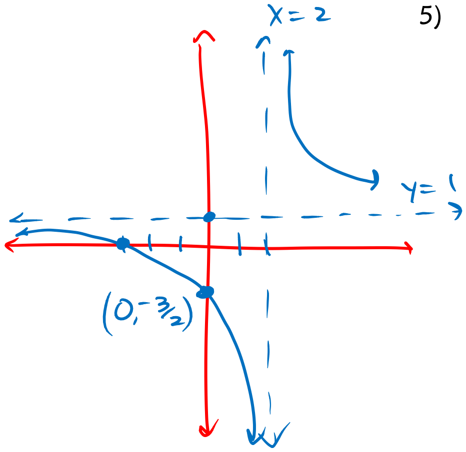
4) $f(x)=\dfrac{x+3}{x-2}$

VA: $x=2$

HA: $y=1$

x int: $x=-3$ $(-3,0)$

y int: $(0,-\frac{3}{2})$



5) $f(x)=\dfrac{-x^2+4x}{(x^2-9)(x-4)}=\dfrac{-x\cancel{(x-4)}}{(x+3)(x-3)\cancel{(x-4)}}$

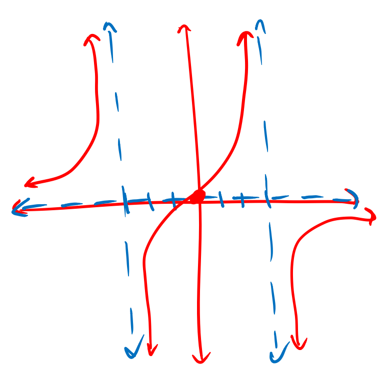
Hole: $x=4$ $\frac{-4}{7(1)}=-\frac{4}{7}$ $(4,-\frac{4}{7})$

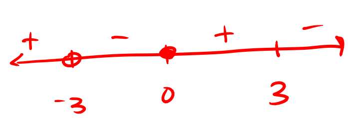
VA: $x=3$ $x=-3$

HA: $y=0$

x int: $(0,0)$

y int: $(0,0)$





Test $\dfrac{-x}{(x+3)(x-3)}$

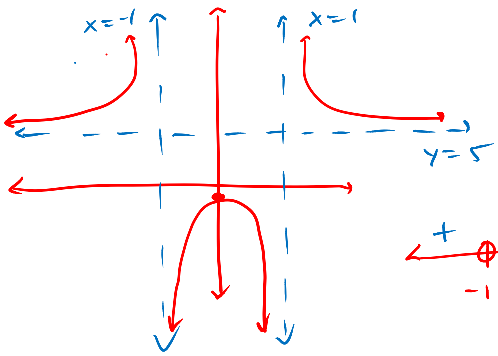
6) $f(x) = \dfrac{5x^2+1}{x^2-1} = \dfrac{5x^2+1}{(x+1)(x-1)}$

VA: $x=-1$ $x=1$

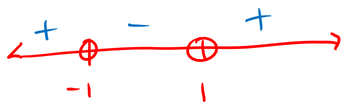
HA: $y=5$

x int: no real → $5x^2+1=0$, $x^2=-\frac{1}{5}$, $x=\pm\sqrt{-\frac{1}{5}}$

y int: $(0,-1)$



Test: $\dfrac{5x^2+1}{(x+1)(x-1)}$



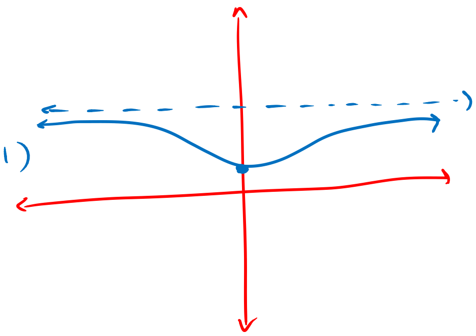
7) $f(x) = \dfrac{5x^2+1}{x^2+1}$

VA: none!

HA: $y=5$

x int: none

y int: $(0,1)$



TRY NUMBERS 8 AND 9 ON YOUR OWN AND CHECK WITH THE KEY!
I'LL GO OVER THESE WITH YOU TOMORROW IF YOU STILL HAVE QUESTIONS.

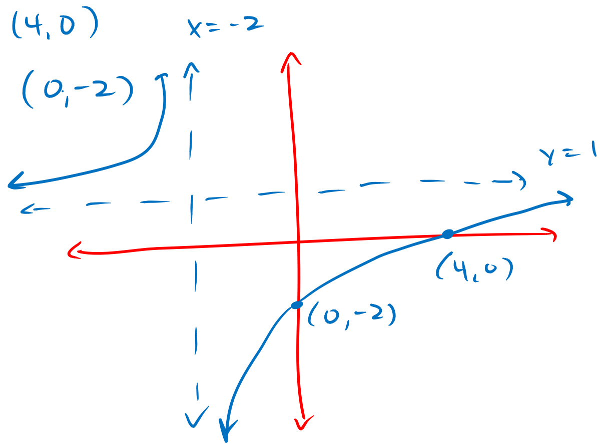
8) $f(x) = \dfrac{x-4}{x+2}$

VA: $x=-2$

HA: $y=1$

x int: $(4,0)$

y int: $(0,-2)$



9) $f(x) = \dfrac{3}{x^2+2}$

VA: none

HA: $y=0$

x int: none

y int: $(0, \frac{3}{2})$