

8.3 SIMPLIFYING RADICALS

OBJECTIVE:

- Add, subtract, multiply, divide and rationalize radical expressions.

You are already used to simplifying square roots. Now, we are going to increase the difficulty by asking you to simplify an n^{th} root.

Simplify: 1, 4, 9, 16, 25, 36, ...

$$1. \sqrt{48} = \sqrt{16 \cdot 3} = \boxed{4\sqrt{3}}$$

1, 2³, 3³, 4³, 5³, 6³, ...

$$2. \sqrt[3]{72} = \sqrt[3]{8 \cdot 9} = \boxed{2\sqrt[3]{9}}$$

1, 8, 27, 64, 125, 216, ...

$$3. 4\sqrt{125} - 8\sqrt{20} + 2\sqrt{180}$$

$$\begin{aligned} &4 \cdot \sqrt{25} \sqrt{5} - 8 \sqrt{4} \sqrt{5} + 2 \sqrt{36} \sqrt{5} \\ &4 \cdot 5 \sqrt{5} - 8 \cdot 2 \sqrt{5} + 2 \cdot 6 \sqrt{5} \\ &20 \sqrt{5} - 16 \sqrt{5} + 12 \sqrt{5} \\ &\boxed{16 \sqrt{5}} \end{aligned}$$

$$4. 2\sqrt[3]{48} - \sqrt[3]{384} + 3\sqrt[3]{750}$$

$$\begin{aligned} &2\sqrt[3]{8} \sqrt[3]{6} - \sqrt[3]{64} \sqrt[3]{6} + 3\sqrt[3]{125} \sqrt[3]{6} \\ &2 \cdot 2 \sqrt[3]{6} - 4 \sqrt[3]{6} + 3 \cdot 5 \sqrt[3]{6} \\ &4 \sqrt[3]{6} - 4 \sqrt[3]{6} + 15 \sqrt[3]{6} \\ &\boxed{15 \sqrt[3]{6}} \end{aligned}$$

RATIONALIZING THE DENOMINATOR:

$$5. \sqrt{\frac{16}{5}}$$

$$\frac{\sqrt{16}}{\sqrt{5}} = \frac{4}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \boxed{\frac{4\sqrt{5}}{5}}$$

$$6. \frac{4}{\sqrt[3]{5}}$$

$$\frac{4}{\sqrt[3]{5}} \cdot \frac{\sqrt[3]{5}}{\sqrt[3]{5}} \quad \text{no!}$$

$$\begin{aligned} \frac{4}{\sqrt[3]{5}} \cdot \frac{\sqrt[3]{5^2}}{\sqrt[3]{5^2}} &= \frac{4\sqrt[3]{5^2}}{\sqrt[3]{5^3}} \\ &= \boxed{\frac{4\sqrt[3]{25}}{5}} \end{aligned}$$

$$7. \frac{1}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}} = \boxed{\frac{\sqrt[3]{4}}{2}}$$

$$8. \frac{1}{\sqrt[5]{4}} = \frac{1}{\sqrt[5]{2^2}} \cdot \frac{\sqrt[5]{2^3}}{\sqrt[5]{2^3}} = \boxed{\frac{\sqrt[5]{8}}{2}}$$

BIGGER PICTURE:

Why does $\sqrt{36} = 6$?



INVERSES!

$$\sqrt{36} = \sqrt[2]{6^2} = \sqrt[2]{6^2} = 6$$

So, if $\sqrt[3]{27} = \sqrt[3]{3^3} = 3$, then

$$\sqrt[x]{y^x} = y$$

$$9. \frac{4}{\sqrt{7} + \sqrt{3}} \cdot \frac{\sqrt{7} - \sqrt{3}}{\sqrt{7} - \sqrt{3}} = \frac{4\sqrt{7} - 4\sqrt{3}}{7 - 3} = \frac{4\sqrt{7} - 4\sqrt{3}}{4} = \boxed{\sqrt{7} - \sqrt{3}}$$

RECALL THESE BASIC PATTERNS:

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x + y)(x - y) = x^2 - y^2$$

$$10. \frac{3}{2\sqrt{6}} - \frac{3\sqrt{6}}{6}$$

$$\frac{3}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} - \frac{3\sqrt{6}}{6} =$$

$$\frac{3\sqrt{6}}{12} - \frac{3\sqrt{6}}{6} =$$

$$\frac{3\sqrt{6}}{12} - \frac{6\sqrt{6}}{12} = \frac{-3\sqrt{6}}{12} = \boxed{\frac{-\sqrt{6}}{4}}$$

$$11. \frac{6}{\sqrt[4]{9}} + 2\sqrt[4]{3}$$

$$\frac{6}{\sqrt[4]{3^2}} + 2\sqrt[4]{3}$$

$$\frac{6}{\sqrt[2]{3}} + 2\sqrt[4]{3}$$

$$\frac{6 \cdot \sqrt{3}}{\sqrt{3} \sqrt{3}} + 2\sqrt[4]{3}$$

$$\frac{6\sqrt{3}}{3} + 2\sqrt[4]{3}$$

$$\boxed{2\sqrt{3} + 2\sqrt[4]{3}}$$

COMMON ERRORS!

$$1) \frac{4}{7 + \sqrt{2}} = \frac{4}{7 + \sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{7\sqrt{2} + 2} = \frac{4\sqrt{2}}{9}$$

$$2) \frac{4}{\sqrt[3]{3}} = \frac{4}{\sqrt[3]{3}} \cdot \frac{\sqrt[3]{3}}{\sqrt[3]{3}} = \frac{4\sqrt[3]{3}}{3}$$

$$1) \frac{4}{7 + \sqrt{2}} \cdot \frac{7 - \sqrt{2}}{7 - \sqrt{2}} = \frac{28 - 4\sqrt{2}}{49 - 2} = \boxed{\frac{28 - 4\sqrt{2}}{47}}$$

$$2) \frac{4}{\sqrt[3]{3}} \cdot \frac{\sqrt[3]{3^2}}{\sqrt[3]{3^2}} = \boxed{\frac{4\sqrt[3]{9}}{3}}$$