480/81
Day 1
Notes

7.9 Notes – Graphing Rational Functions

Objectives:

1) Determine vertical and horizontal asymptotes and x and y intercepts to graph rational functions.

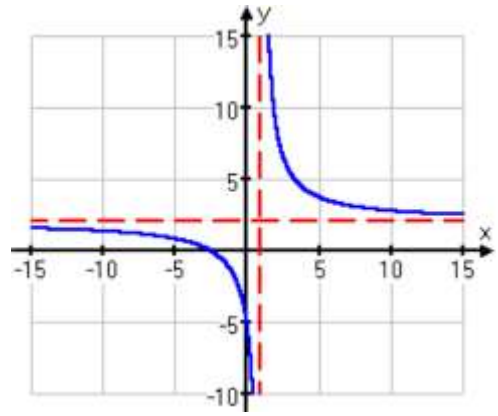
Rational Functions:

A rational function is a function of the form $y = \dfrac{f(x)}{g(x)}$,

where $f(x)$ and $g(x)$ are polynomials.

A rational function can have both vertical AND horizontal asymptotes.

A rational function has branches – one more branch than the number of vertical asymptotes.



Graph above: one vertical asymp. and 2 branches

Finding Vertical Asymptotes:

To find the vertical asymptotes, set the denominator equal to 0 and solve for x. There may be more than one!

1) $y = \dfrac{x-3}{x^2-9}$

$y = \dfrac{\cancel{x-3}}{(x+3)\cancel{(x-3)}}$

$x = -3$ $\quad$ $x = \cancel{3}$ ← hole $\quad$ 2 branches

2) $y = \dfrac{x^2-4}{x^2+4x-5}$

$y = \dfrac{(x+2)(x-2)}{(x+5)(x-1)}$ $\quad$ $x = -5$ $\quad$ $x = 1$ $\quad$ 3 branches

3) $y = \dfrac{x^2}{x+2}$

$y = \dfrac{x^2}{x+2}$ $\quad$ $x = -2$ $\quad$ 2 branches

4) $y = \dfrac{1}{x^2+2}$

$x^2 + 2 = 0$ $\quad$ NO VERTICAL ASYMPTOTES!

HOW MANY BRANCHES WOULD EACH GRAPH HAVE?

FINDING HORIZONTAL ASYMPTOTES:

To find the horizontal asymptote, compare the degree of the numerator(n) to the degree of the denominator(d).

There are three different cases: (The maximum number of horizontal asymptotes is one.)

1) If $n > d$ ⟶ No horizontal asymptote! Ex. $f(x) = \dfrac{-3x^3 + x - 1}{x - 2}$

If the degree of the numerator is **GREATER** than the degree of the denominator, there is no horizontal asymptote.

2) If $n = d$ ⟶ $y = \dfrac{\text{lead coefficient of numerator}}{\text{lead coefficient of denominator}}$ Ex. $f(x) = \dfrac{4x^2 + 1}{x^2 - 7}$

If the degree of the numerator is **EQUAL** to the degree of the denominator, the asymptote is found by dividing the coefficients of the terms with the highest degrees.

3) If $n < d$ ⟶ y = 0 Ex. $f(x) = \dfrac{2x + 2}{x^2 - 3}$

If the degree of the numerator is **LESS THAN** the degree of the denominator, the horizontal asymptote is at y = 0.

1) **"IF YOU BIG ON TOP, JUST STOP!" (CUZ THEY AIN'T NO HORIZONTAL ASYMPTOTE)**

2) Got nothin' for this one. Ain't nobody got time for that.

3) **"IF THE BOTTOM IS HEAVY, THE LINE IS STEADY (AT $Y = 0$)".**

AN EXAMPLE OF A RATIONAL FUNCTION: $f(x) = \dfrac{x^2 - 3x + 2}{x^2 - 6x + 8}$

1) Factor the numerator and denom.

2) Find any holes.
Hole - occurs when factors cancel

3) Find the x-intercept(s). Let y = 0.

4) Find the y-intercept. Let x = 0.

5) Find all vertical asymptotes.
Set denominator = 0.

6) Find the horizontal asymptote.

7) Sketch the branches of the curve.

1) $f(x) = \dfrac{x^2 - 3x + 2}{x^2 - 6x + 8} = \dfrac{(x-2)(x-1)}{(x-2)(x-4)}$

2) $f(x) = \dfrac{\cancel{(x-2)}(x-1)}{\cancel{(x-2)}(x-4)}$ Hole: x = 2. Find f(2): $f(2) = \dfrac{(2-1)}{(2-4)} = \dfrac{1}{-2}$

HOLE: $\left(2, \dfrac{1}{-2}\right)$

3) x – int: $0 = \dfrac{(x-1)}{(x-4)}$

$0 = (x-1)$

$x = 1$

4) y – int: $y = \dfrac{(0-1)}{(0-4)}$

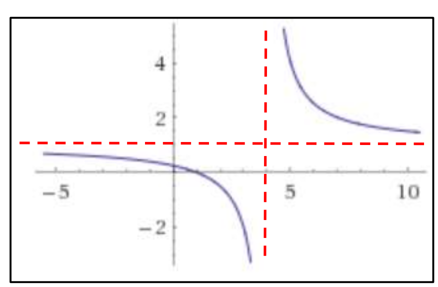
$y = \dfrac{1}{4}$

5) VA: $0 = (x-4)$ (denominator = 0)

VA: $x = 4$

6) HA: Degree is same! $y = \dfrac{1x^2}{1x^2} = 1$ HA: $y = 1$

7)



STEPS FOR GRAPHING:

1) Factor the numerator and denom.

2) Find any holes.
   Hole - occurs when factors cancel

3) Find the x-intercept(s). Let y = 0.

4) Find the y-intercept. Let x = 0.

5) Find all vertical asymptotes.
   Set denominator = 0.

6) Find the horizontal asymptote.

7) Sketch the branches of the curve.

EX 1) $f(x) = \dfrac{4}{x-5}$

x int: set $0 = \dfrac{4}{x-5}$

$0 = 4$

DNE

y int set $x=0$

$y = \dfrac{4}{-5}$

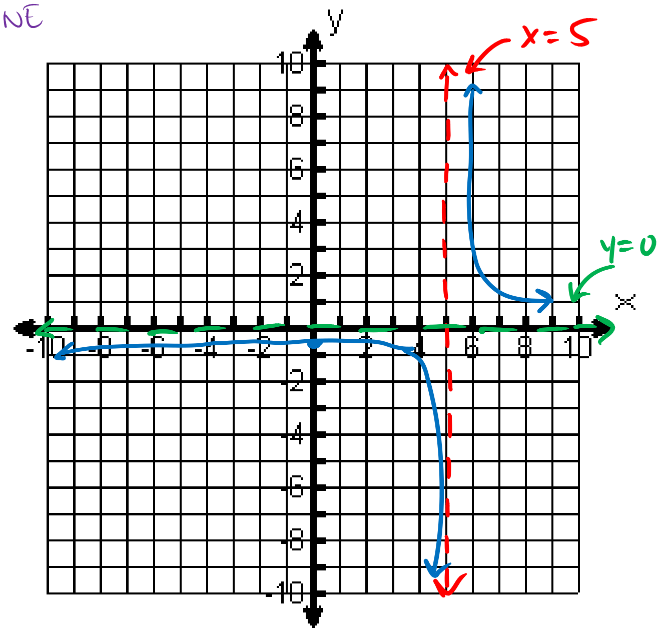
hole(s): none

x – int: DNE

y – int: $\left(0, -\frac{4}{5}\right)$

VA: $x=5$

HA: $y=0$



EX 2) $f(x) = \dfrac{x^2+3x+2}{x^2+x-2}$

$$\frac{(x+1)\cancel{(x+2)}}{\cancel{(x+2)}(x-1)}$$

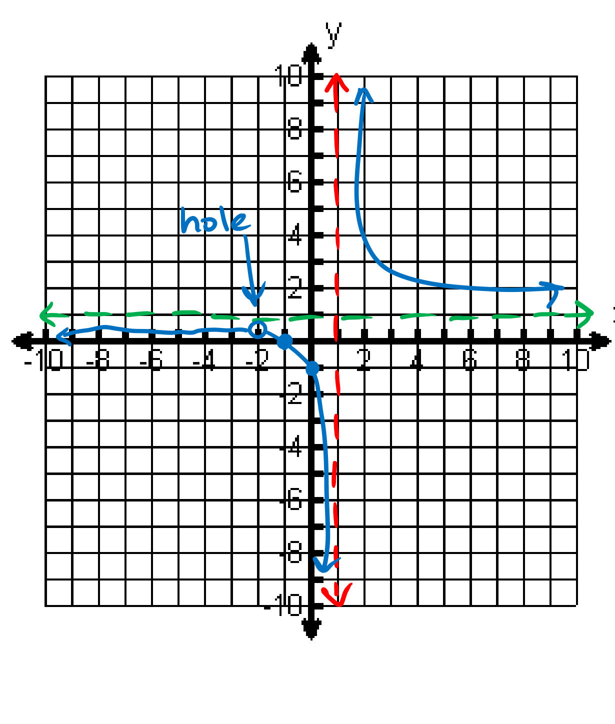
hole(s): $\left(-2, \frac{1}{3}\right)$

x – int: $(-1, 0)$

y – int: $(0, -1)$

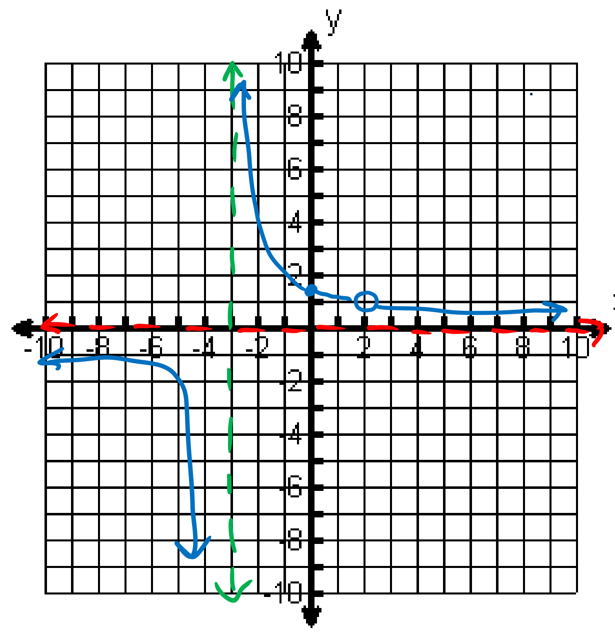
VA: $x=1$

HA: $y=1$



3. $f(x) = \dfrac{5x-10}{x^2+x-6}$

$\dfrac{5\cancel{(x-2)}}{(x+3)\cancel{(x-2)}}$ $f(x) = \dfrac{5}{x+3}$



hole(s): $(2, 1)$

x – int: DNE

y – int: $(0, 5/3)$

VA: $x = -3$

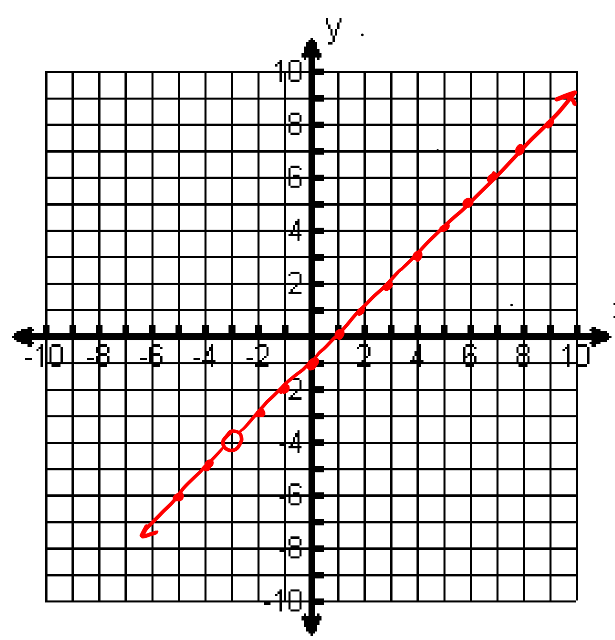
HA: $y = 0$

4. $f(x) = \dfrac{x^2+2x-3}{x+3}$

$f(x) = \dfrac{\cancel{(x+3)}(x-1)}{\cancel{x+3}}$

$f(x) = x-1$

$0 = x-1$ $x = 1$



hole(s): $(-3, -4)$

x – int: $(1, 0)$

y – int: $(0, -1)$

VA: DNE

HA: DNE