4.1 Notes
Day 1

SOLVE LINEAR SYSTEMS BY SUBSTITUTION AND ELIMINATION

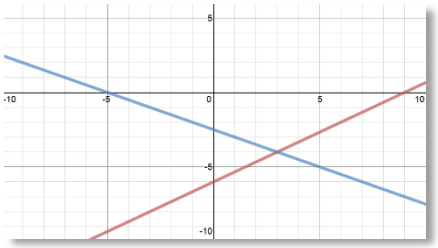
Name KEY
Per ____ Date ____

A SYSTEM OF EQUATIONS:

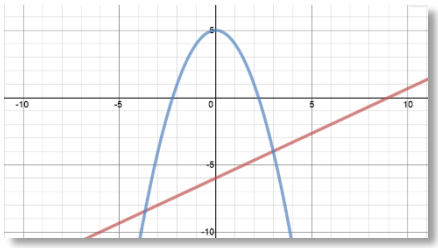
Definition: A system of equations is a **collection of two or more equations with a same set of unknowns**. In solving a system of equations, we try to find values for each of the unknowns that will satisfy every equation in the system.

The equations in the system can be linear or non-linear. We focus on linear equations in Ch 4.

$\begin{cases} -2x+3y=-6 \\ x+2y=-5 \end{cases}$



$\begin{cases} -2x+3y=-6 \\ x^2+y=5 \end{cases}$



SOLVING A LINEAR SYSTEM BY SUBSTITUTION

Solve the following systems algebraically.

1) $\begin{cases} -2x-3y=18 \\ 6x-5y=12 \end{cases}$

$-2x=3y+18$

$x=-\frac{3}{2}y-9$ ← now substitute for x

$6\left(-\frac{3}{2}y-9\right)-5y=12$

$-9y-54-5y=12$

$-14y=66$

$y=\frac{-33}{7}$ (substitute again)

$-2x-3\left(\frac{-33}{7}\right)=18$

$-2x+\frac{99}{7}=18$

$-2x=\frac{126}{7}-\frac{99}{7}$

$\frac{-2x}{-2}=\frac{27}{7}\div -2$ is same as $\cdot\frac{-1}{2}$

$x=-\frac{27}{14}$

$\left(-\frac{27}{14},-\frac{33}{7}\right)$

2) $\begin{cases} y=6x-4 \\ y=-x-5 \end{cases}$

Set equal!

$6x-4=-x-5$

$7x=-1$

$x=-\frac{1}{7}$

$y=6\left(-\frac{1}{7}\right)-4$

$y=-\frac{6}{7}-\frac{28}{7}=-\frac{34}{7}$

$\left(-\frac{1}{7},-\frac{34}{7}\right)$

3) $\begin{cases} 3x-6y=12 \\ x-2y=4 \end{cases}$

$x=2y+4$

$3(2y+4)-6y=12$

$6y+12-6y=12$

$12=12$ or $0=0$ → True!

Same line! Infinite points

Answer: ∞ points (or ALL) on the line $x-2y=4$

4) $\begin{cases} 3x-6y=12 \\ x-2y=6 \end{cases}$

$x=2y+6$

$3(2y+6)-6y=12$

$6y+18-6y=12$

$18=12$ or $6=0$ → False!

These never intersect. (parallel)

Answer: No solution Parallel lines!

SOLVING A LINEAR SYSTEM BY ELIMINATION

Solve the following systems algebraically. Start off easy:

1) $\begin{cases} -4x - 2y = -12 \\ 4x + 8y = -24 \end{cases}$ Add these lines to eliminate the "x"

$6y = -36$

$y = -6$ ← substitute into either equation

$-4x - 2(-6) = -12$

$-4x + 12 = -12$

$-4x = -24$

$x = 6$

$(6, -6)$

2) $\begin{cases} -4x + 9y = 9 \\ (x - 3y = -6)\,4 \end{cases}$ Not ready for elimination yet (mult. bottom by 4)

$4x - 12y = -24$

$-4x + 9y = 9$

$-3y = -15$

$y = 5$

$x - 3(5) = -6$

$x - 15 = -6$

$x = 9$

Solution: $(9, 5)$

3) $\begin{cases} (3x - 6y = 12)\,4 \\ (4x - 8y = 48)\,-3 \end{cases}$

$12x - 24y = 48$

$-12x + 24y = -144$

$0 = -96$

False!

No solution, parallel lines

4) $\begin{cases} \frac{4}{x} - \frac{3}{y} = 11 \\ \frac{5}{x} - \frac{6}{x} = 9 \end{cases}$ $\begin{cases} 4\left(\frac{1}{x}\right) - 3\left(\frac{1}{y}\right) = 11 \\ 5\left(\frac{1}{x}\right) - 6\left(\frac{1}{y}\right) = 9 \end{cases}$

Let $a = \frac{1}{x}$ $b = \frac{1}{y}$

New system $\begin{cases} (4a - 3b = 11)\,-2 \\ 5a - 6b = 9 \end{cases}$ multiply by -2

$-8a + 6b = -22$

$5a - 6b = 9$

$-3a = -13$

$a = \frac{13}{3}$

$5\left(\frac{13}{3}\right) - 6b = 9$

$\frac{65}{3} - 6b = 9$

$-6b = \frac{27}{3} - \frac{65}{3}$

$-6b = \frac{-38}{3}$

$b = \frac{-38}{18}$ $b = -\frac{19}{9}$