BC CALC: DAY 1 REVIEW OF FUNCTIONS

OBJECTIVES: 1) Review many topics pertaining to functions.
2) Test the symmetry of functions.

FINDING THE DOMAIN OF A FUNCTION

1) Find the domain of f: $f(x)=\dfrac{\sqrt{4+x}}{1-x}$

$4+x\geq 0$

$x\geq -4$

$x\neq 1$

D: $[-4,1)\cup(1,\infty)$

COMPOSITE FUNCTIONS

For every composite function $f(g(x))$: the domain of $f(g(x))$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f.

If $f(x)=x^2-16$ and $g(x)=\sqrt{x}$ find:

2) $f\circ g$ and its domain.

$f(g(x))=\sqrt{x}^2-16$

$=x-16$

D: $\mathbb{R}$? NO! $\Rightarrow$ only

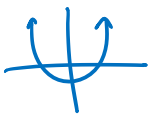
$x\geq 0$

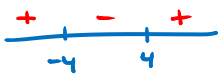
3) $g\circ f$ and its domain.

$g(f(x))=\sqrt{x^2-16}$

$x^2-16\geq 0$

$(x+4)(x-4)\geq 0$





$(-\infty,-4]\cup[4,\infty)$

THE DIFFERENCE QUOTIENT

4) Find $\dfrac{f(x+h)-f(x)}{h}$ for $f(x)=x^2-5x+1$.

$f(x+h)=(x+h)^2-5(x+h)+1$

$f(x)=x^2-5x+1$

$\dfrac{x^2+2xh+h^2-5x-5h+1-(x^2-5x+1)}{h}$

$\dfrac{2xh+h^2-5h}{h}=\boxed{2x+h-5}$

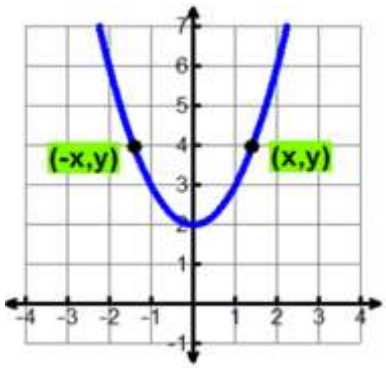
5) Find $\dfrac{f(x)-f(a)}{x-a}$ for $f(x)=\dfrac{1}{3x}$.

$\dfrac{\frac{1}{3x}-\frac{1}{3a}}{x-a}\cdot\dfrac{3ax}{3ax}$

$\dfrac{a-x}{3ax(x-a)}=\dfrac{-(x-a)}{3ax(x-a)}=\boxed{\dfrac{-1}{3ax}}$

EVEN VS. ODD FUNCTIONS

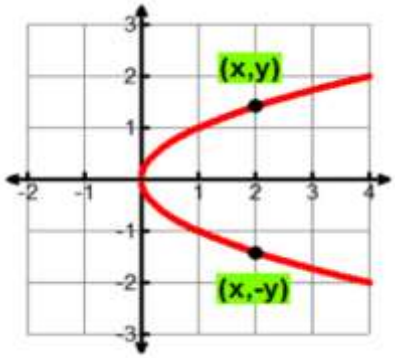
Symmetric with respect to the y-axis



EVEN

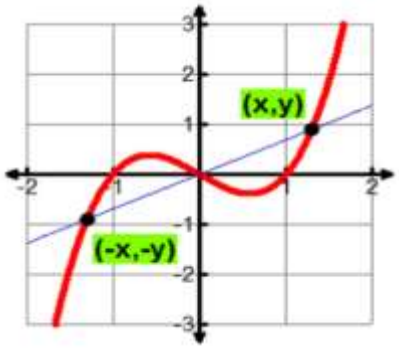
TEST: Plug in –x.

Symmetric with respect to the x-axis



TEST: Plug in –y.

Symmetric about the origin

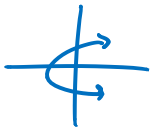


ODD

TEST: Plug in –x and –y.

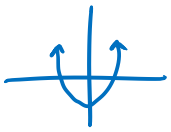
6) Determine the symmetry of the graph of each equation.

a. $y^2 = x + 4$



Symm. w.r.t. x-axis

b. $y = x^2 - 2$

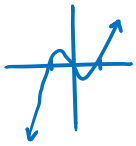


Sym. w.r.t. y-axis

c. $y = x^3 - 4x$

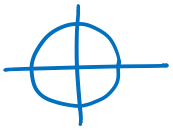
$x(x^2-4)$

$x(x+2)(x-2)$



Symm. w.r.t. origin

d. $x^2 + y^2 = 4$



Symm. w.r.t. x & y axis and origin

7) Determine whether each function is even, odd, or neither.

a. $y = x^3 - 4x$

$-y = (-x)^3 - 4(-x)$

$-y = -x^3 + 4x$

$y = x^3 - 4x$ ✓

ODD

b. $y = x^2 - 2$

$-y = (-x)^2 - 2$

$-y = x^2 - 2$

$y = -x^2 + 2$

EVEN!

$y = (-x)^2 - 2$

$y = x^2 - 2$

c. $x^2 + y^2 = 4$

NONE!

not a function

FAMILY OF FUNCTIONS!

TRANSFORMATIONS

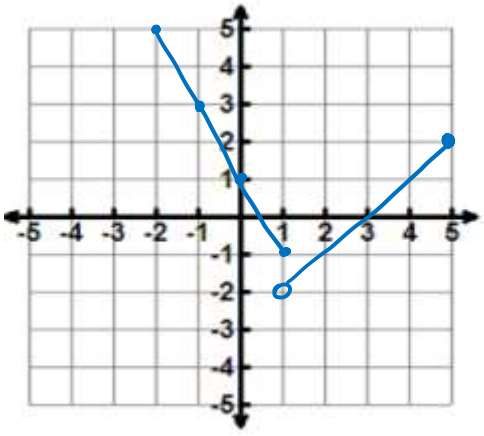
The parent function is $f(x)$:

- The graph of $y = f(x) + k$ is shifted **k** units **upward.**
- The graph of $y = f(x) - k$ is shifted **k** units **downward.**
- The graph of $y = f(x + h)$ is shifted **h** units to the **left.**
- The graph of $y = f(x - h)$ is shifted **h** units to the **right.**
- The graph of $y = -f(x)$ is reflected over the **x-axis.**
- The graph of $y = f(-x)$ is reflected over the **y-axis.**

PIECEWISE FUNCTIONS

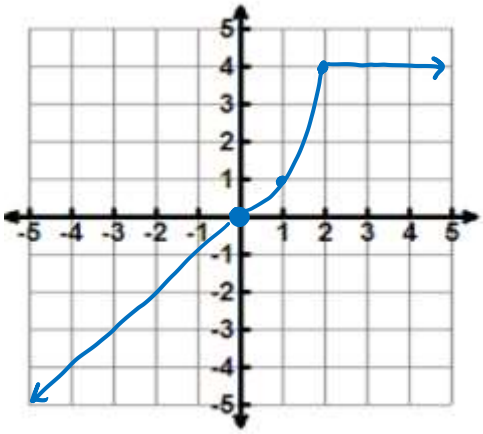
Like the name says these functions are graphed in pieces or in parts.

8) Graph $g(x)=\begin{cases}-2x+1, & -2\le x\le 1\\ x-3, & 1<x\le 5\end{cases}$



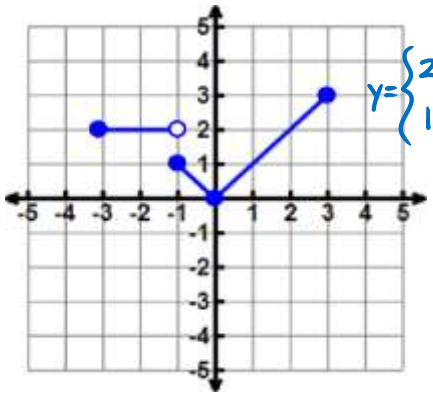
Domain: $[-2,5]$ Range: $(-2,5]$

9) Graph $f(x)=\begin{cases}-x, & x<0\\ x^2, & 0\le x\le 2\\ 4, & x>2\end{cases}$



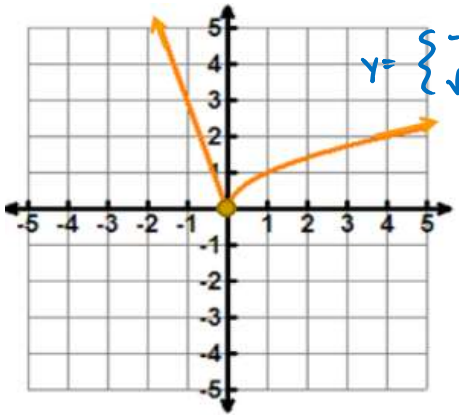
Domain: $(-\infty,\infty)$ Range: $(-\infty,4]$

10) Write the equation of the function.



$y=\begin{cases}2, & -3\le x<-1\\ |x| & -1\le x\le 3\end{cases}$

11) Write the equation of the function.



$y=\begin{cases}-3x, & x\le 0\\ \sqrt{x}, & x\ge 0\end{cases}$

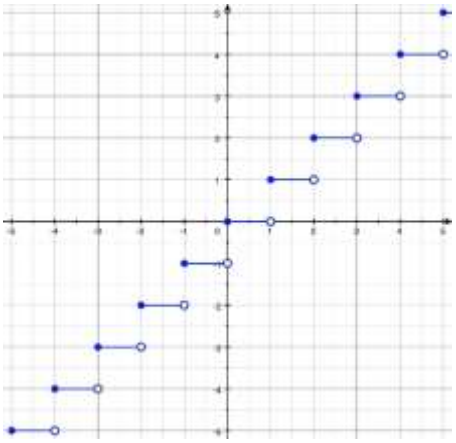
THE GREATEST INTEGER FUNCTION

(Also called the Floor Function or the Step Function.)

$y = x$

Domain: $(-\infty, \infty)$

Range: Integers $\mathbb{Z}$



12) Find the following values:

a) 2.9 2

b) .5 0

c) −2.9 −3

d) −.5 −1