

7.9 NOTES – GRAPHING RATIONAL FUNCTIONS

Graph each of the following rational functions.

$$1. f(x) = \frac{5}{(x-1)^2(x+3)}$$

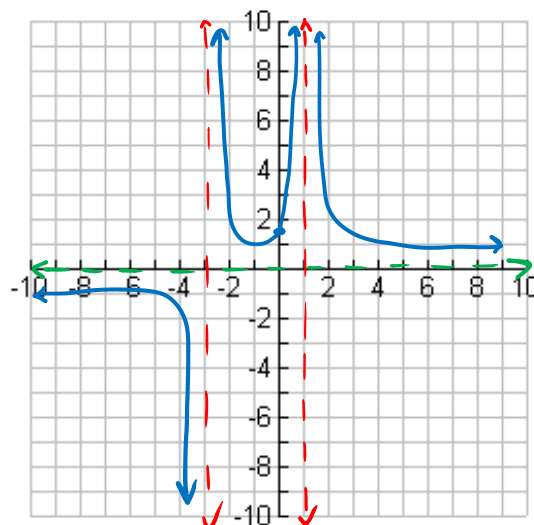
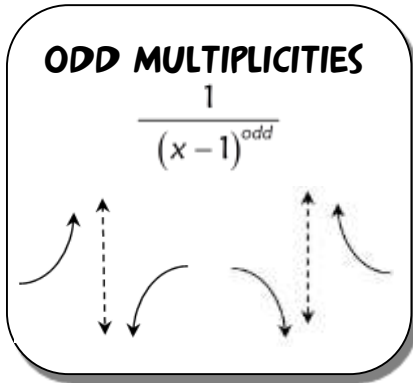
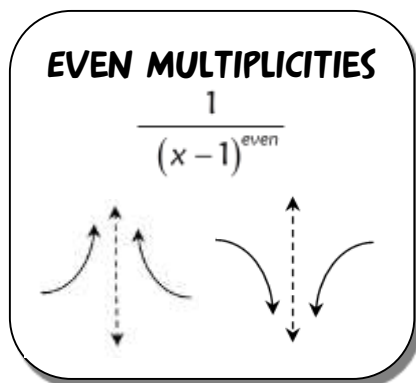
hole(s): DNE

x – int: DNE

y – int: $(0, 5/3)$

VA: $x=1$ $x=-3$

HA: $y=0$



$$2. f(x) = \frac{(x+4)(\cancel{x-1})}{(x-1)^2(\cancel{x+4})} = \frac{1}{x-1}$$

Hole $x=-4$

$$f(-4) = \frac{1}{5}$$

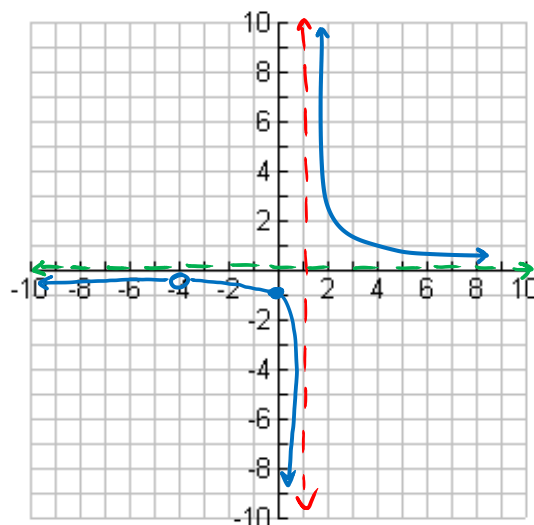
hole(s): $(-4, \frac{1}{5})$

x – int: DNE

y – int: $(0, -1)$

VA: $x=1$

HA: $y=0$



$$3. f(x) = \frac{x^3 + 8x^2 + 9x - 18}{x + 6}$$

$$\begin{array}{r|rrrr} -6 & 1 & 8 & 9 & -18 \\ & & -6 & -12 & 18 \\ \hline & 1 & 2 & -3 & 0 \end{array}$$

$$f(x) = \frac{(x-1)(x+3)(x+6)}{(x+6)}$$

$$x^2 + 2x - 3$$

$$(x-1)(x+3)$$

$$f(-6) = (-7)(-3) = 21$$

Quadratic!

Find vertex

$$\frac{-2}{2(1)} = -1$$

$$v: (-1, -4)$$

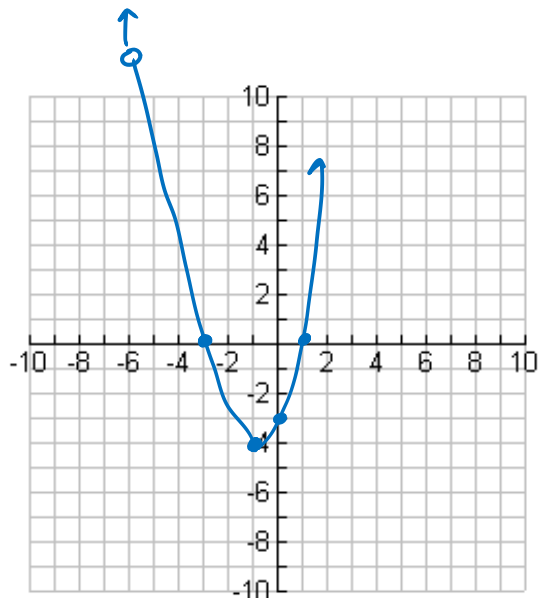
hole(s): $(-6, 21)$

x-int: $(1, 0)$ $(-3, 0)$

y-int: $(0, -1)$

VA: NONE

HA: NONE



$$4. f(x) = \frac{(x-2)}{x^3 + 5x^2 - 8x - 12}$$

$$\begin{array}{r|rrrr} 2 & 1 & 5 & -8 & -12 \\ & & 2 & 14 & 12 \\ \hline & 1 & 7 & 6 & 0 \end{array}$$

$$f(x) = \frac{(x-2)}{(x-2)(x+6)(x+1)}$$

$$x^2 + 7x + 6$$

$$(x+6)(x+1)$$

Hole: $x=2$

$$\frac{1}{0(3)} = \frac{1}{24}$$

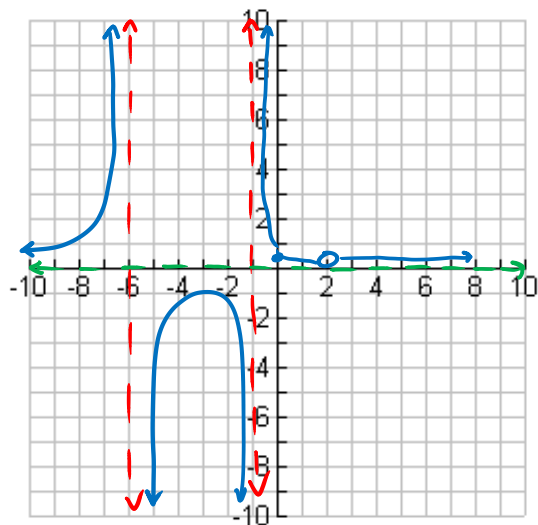
hole(s): $(2, \frac{1}{24})$

x-int: DNE

y-int: $(0, \frac{1}{6})$

VA: $x = -1$ $x = -6$

HA: $y = 0$



GOING THROUGH THE HORIZONTAL ASYMPTOTE

$$5. f(x) = \frac{(x-4)(x+1)}{(x+4)(x-1)^2}$$

$$f(2) = \frac{(-2)(3)}{(6)(1)} = -1$$

hole(s): **None**

x-int: **(4,0)(-1,0)**

y-int: **(0,-1)**

VA: **$x=1$ $x=-4$**

HA: **$y=0$**

$$6. f(x) = \frac{2x^2 - 2x - 24}{x^2 + x - 12}$$

$$\frac{2(x^2 - x - 12)}{x^2 + x - 12} = \frac{2(x+3)(x-4)}{(x+4)(x-3)}$$

$$f(0) = \frac{2(3)(-4)}{(4)(-3)} = 2$$

hole(s): **NONE**

x-int: **(-3,0)(4,0)**

y-int: **(0,2)**

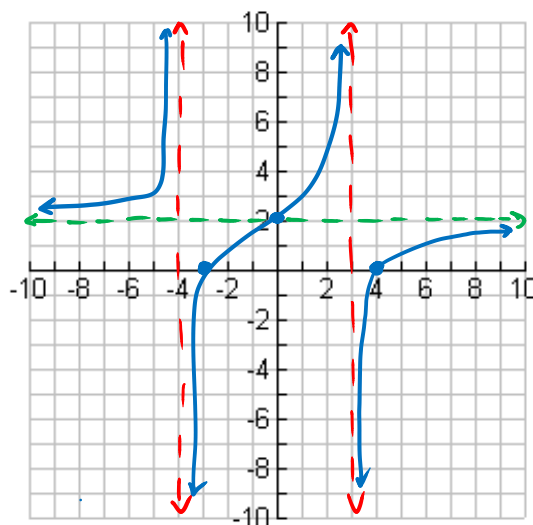
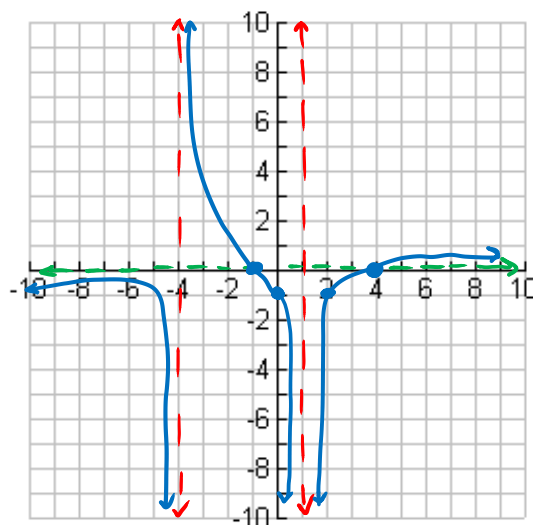
VA: **$x=3$ $x=-4$**

HA: **$y=2$**

Functions can **never cross a vertical asymptote**, but sometimes, they can cross the horizontal asymptote.

HORIZONTAL ASYMPTOTE:

- Tells the "long run" behavior of the function past the vertical asymptotes or x-intercepts.
- Horizontal asymptotes are not asymptotic in the middle. It is okay to cross a horizontal asymptote in the middle.



7. $f(x) = \frac{16}{x^2 + 4}$

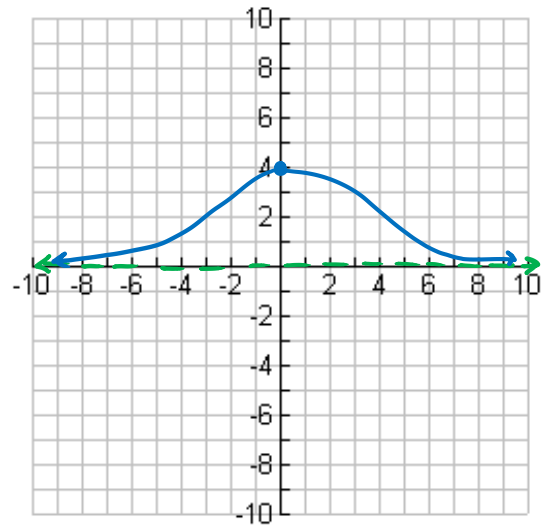
hole(s): **NONE**

x - int: **DNE**

y - int: **(0, 4)**

VA: **DNE**

HA: **$y = 0$**



8. $f(x) = \frac{5x^2}{x^2 + 2}$

hole(s): **NONE**

x - int: **(0, 0)**

y - int: **(0, 0)**

VA: **DNE**

HA: **$y = 5$**

