8.4 NOTES – RADICAL EQUATIONS

OBJECTIVES:
1) Solve a radical equation and recognize when to discard extraneous solutions.

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RADICAL EQUATION—is an equation in which a variable appears under a radical sign:
$$x = 3 + \sqrt{24 - 4x} \qquad 4\sqrt{x+1} = x+1 \qquad \sqrt{x-3} = \sqrt{x+4}$$

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STEPS FOR SOLVING RADICAL EQUATIONS:
1.) State the Domain.
2.) Isolate the radical—Get the radical by itself on one side of the equation.
3.) Square, cube, etc, each member of the equation.
      Are there any radicals remaining? If so, go back to step 1.
4.) Solve the polynomial equations. You might have to factor or use the quadratic formula.
5.) Check for any extraneous solutions. Write the solution set { }.

Solve the radical equation using the steps above.

1.) $3 = x + \sqrt{x-1}$ $x \geq 1$

$3 - x = \sqrt{x-1}$

$9 - 6x + x^2 = x - 1$

$x^2 - 7x + 10 = 0$

$(x-5)(x-2) = 0$

     $x = \cancel{5}, 2$ $\{2\}$

2.) $6 + 3\sqrt{2x} = 0$ $x \geq 0$

$3\sqrt{2x} = -6$

$\sqrt{2x} = -2$

     NO SOLUTION!

     $\{ \}$ $\varnothing$

3.) $\sqrt{x+4} + \sqrt{x-4} = 4$ $x \geq 4$ $x \geq 4$ $x \geq -4$

$\sqrt{x+4} = \left(4 - \sqrt{x-4}\right)^2$

$x + 4 = 16 - 8\sqrt{x-4} + x - 4$

$-8 = -8\sqrt{x-4}$

$1 = \sqrt{x-4}$

$1 = x - 4$

$x = 5$ $\{5\}$

4.) $\sqrt[3]{3x-1} = 2$

$3x - 1 = 8$

$3x = 9$

$x = 3$

     $\{3\}$

5.) $2\sqrt{x} - \sqrt{4x-3} = \dfrac{1}{\sqrt{4x-3}}$ $x > \dfrac{3}{4}$

$2\sqrt{4x^2-3x} - (4x-3) = 1$

$2\sqrt{4x^2-3x} = 4x-2$

$\sqrt{4x^2-3x} = 2x-1$

$4x^2-3x = 4x^2-4x+1$

$x = 1 \qquad \{1\}$

6.) $\sqrt{x^2+5x-6} + \sqrt{x^2+3x-3} = 1$

$\sqrt{x^2+5x-6} = 1 - \sqrt{x^2+3x-3}$

$x^2+5x-6 = 1 - 2\sqrt{x^2+3x-3} + x^2+3x-3$

$2x-4 = -2\sqrt{x^2+3x-3}$

$-(x-2) = \sqrt{x^2+3x-3}$

$x^2-4x+4 = x^2+3x-3$

$-7x = -7 \qquad \{1\}$

$x = 1$

7.) $\sqrt{x+2} - \sqrt{x^2+5x+6} = 0$

$x+2 = x^2+5x+6$

$0 = x^2+4x+4$

$(x+2)^2 = 0$

$x = -2 \qquad \{-2\}$

8.) $\dfrac{1}{1-x} + \dfrac{1}{1+\sqrt{x}} = \dfrac{1}{1-\sqrt{x}}$ $x > 0 \quad x \neq 1$

$\dfrac{1}{1-x} + \dfrac{1-\sqrt{x}}{1-x} = \dfrac{1+\sqrt{x}}{1-x}$

$2 - \sqrt{x} = 1 + \sqrt{x}$

$1 - \sqrt{x} = \sqrt{x}$

$1 - 2\sqrt{x} + x = x$

$-2\sqrt{x} = -1$

$\sqrt{x} = \dfrac{1}{2}$

$x = \dfrac{1}{4} \qquad \left\{\dfrac{1}{4}\right\}$