THREE VARIABLES SYSTEMS NOTES

OBJECTIVES: Solve a system with three or more variables.

SOLVE THE SYSTEM OF EQUATIONS ALGEBRAICALLY:

1) $\begin{cases} x+y+z=7 \\ 2x-z=6 \\ z=4 \end{cases}$

$2x-4=6$

$2x=10$

$x=5$

$x+y+z=7$

$5+y+4=7$

$y=-2$

$(5,-2,4)$

2) $\begin{cases} A: 2x-y+2z=15 \\ B: -x+y+z=3 \\ C: 3x-y+2z=18 \end{cases}$

A+B

$2x-y+2z=15$

$-x+y+z=3$

$x+3z=18$

B+C

$-x+y+z=3$

$3x-y+2z=18$

$2x+3z=21$

$x+3z=18$

$-(2x+3z=21)$

$-x=-3$

$x=3$

Plug into original

$3+3z=18$

$z=5$

$(3,1,5)$

$2x-y+2z=15$

$2(3)-y+2(5)=15$

$-y=-1$

$y=1$

3) $\begin{cases} A: 5x+3y+2z=2 \\ B: 2x+y-z=5 \\ C: x+4y+2z=16 \end{cases}$

A

$5x+3y+2z=2$

−3B

$-6x-3y+3z=-15$

$-x+5z=-13$

−4B

$-8x-4y+4z=-20$

C

$x+4y+2z=16$

$-7x+6z=-4$

$(-2,6,-3)$

$\begin{cases} -x+5z=-13 \\ -7x+6z=-4 \end{cases}$

$7x-35z=91$

$-29z=87$

$z=-3$

$2x+y-z=5$

$2(-2)+y+3=5$

$y=6$

$-x+5(-3)=-13$

$-x=2$

$x=-2$

4) $\begin{cases} A: x-3y+3z=-4 \\ B: 2x+3y-z=15 \\ C: 4x-3y-z=19 \end{cases}$

A+B

$x-3y+3z=-4$

$2x+3y-z=15$

$3x+2z=11$

B+C

$2x+3y-z=15$

$4x-3y-z=19$

$6x-2z=34$

$\begin{cases} 3x+2z=11 \\ 6x-2z=34 \end{cases}$

$9x=45$

$x=5$

$3(5)+2z=11$

$15+2z=11$

$2z=-4$

$z=-2$

$2x+3y-z=15$

$2(5)+3y+2=15$

$3y=3$

$y=1$

$(5,1,-2)$

WORD PROBLEM APPLICATIONS

5) A theater group sold a total of 440 tickets for $3940. Each regular ticket costs $5, each premium costs $15, and each elite ticket costs $25. The number of regular tickets was three times the number of premium and elite tickets combined. How many of each type were sold?

let x = # of regular tix
y = # of premium tix
z = # of elite tix

$x + y + z = 440$ (from first sentence)
$5x + 15y + 25z = 3940$ (from first & 2nd sentence)
$x = 3(y + z)$ (from 3rd sentence)

6)
$$\begin{cases} 4x + 2y - 2z = 10 & A \\ 2x + 8y + 4z = 32 & B \\ 30x + 12y - 4z = 24 & C \end{cases}$$

Eliminate "z"

2A + B
$8x + 4y - 4z = 20$
$2x + 8y + 4z = 32$
$10x + 12y = 52$

B + C
$2x + 8y + 4z = 32$
$30x + 12y - 4z = 24$
$32x + 20y = 56$

$5(10x + 12y = 52) \rightarrow 50x + 60y = 260$
$-3(32x + 20y = 56) \rightarrow -96x - 60y = -168$

$-46x = 92$
$x = -2$

$50(-2) + 60y = 260$
$-100 + 60y = 260$
$60y = 360$
$y = 6$

Plug into any Original Equation:
$4x + 2y - 2z = 10$
$4(-2) + 2(6) - 2z = 10$
$-8 + 12 - 2z = 10$
$-2z = 6$
$z = -3$

Solution: $(-2, 6, -3)$

7)
$$\begin{cases} 3x - y - 2z = 4 & A \\ 6x + 4y + 8z = 11 & B \\ 9x + 6y + 12z = -3 & C \end{cases}$$

Eliminate "x"

-2A + B
$-6x + 2y + 4z = -8$
$6x + 4y + 8z = 11$
$6y + 12z = 3$

-3A + C
$-9x + 3y + 6z = -12$
$9x + 6y + 12z = -3$
$9y + 18z = -15$

$-3(6y + 12z = 3)$
$2(9y + 18z = -15)$

$-18y - 36z = -9$
$18y + 36z = -30$
$0 = -39$
FALSE!

No solution!