

6.10 NOTES – PROPERTIES OF LOGS

OBJECTIVES:

- 1) Use the properties of logs to expand and condense a logarithmic expression
- 2) Use the properties of logs to evaluate a logarithmic expression
- 3) Use the properties of logs to solve logarithmic equations

THREE GOLDEN RULES!

PROPERTIES OF EXPONENTS

1) Product of Powers: $x^a \cdot x^b = x^{a+b}$

2) Quotient of Powers: $\frac{x^a}{x^b} = x^{a-b}$

3) Power of a Power: $(x^a)^b = x^{ab}$

PROPERTIES OF LOGARITHMS

1) Log of a Product: $\log_x ab = \log_x a + \log_x b$

2) Log of a Quotient: $\log_x \frac{a}{b} = \log_x a - \log_x b$

3) Log of a Power: $\log_x a^b = b \log_x a$

EVALUATE!

Given $\log_b 2 = A$, $\log_b 5 = B$, and $\log_b 7 = C$, evaluate the following:

1) $\log_b \sqrt[5]{4} = \log_b \left(\frac{4}{10}\right)^{\frac{1}{5}}$

$$\frac{1}{5} \log_b \frac{2}{5} = \frac{1}{5} (\log_b 2 - \log_b 5)$$

$$= \boxed{\frac{1}{5} (A - B)}$$

2) $\log_b \left(\frac{7}{16}\right) = \log_b 7 - \log_b 16$

$$\log_b 7 - \log_b 2^4 = \log_b 7 - 4 \log_b 2$$

$$= \boxed{C - 4A}$$

3) $\log_b 3.5 = \log_b \frac{7}{2}$

$$= \log_b 7 - \log_b 2$$

$$= \boxed{C - A}$$

Find the given logarithm WITHOUT using the log key on your calculator. Assume the $\log 3 \approx .477$ and the $\log 7 \approx .845$.

4) $\log 21$

$$\log(3 \cdot 7) = \log 3 + \log 7$$

$$\approx .477 + .845$$

$$\approx \boxed{1.322}$$

5) $\log \left(\frac{7}{3}\right) = \log 7 - \log 3$

$$\approx .845 - .477$$

$$= \boxed{\approx .368}$$

6) $\log 343$

$$\log 7^3$$

$$3 \log 7$$

$$\approx 3(.845)$$

$$\approx \boxed{2.535}$$

EXPAND!

Expand and simplify using the properties of logarithms:

7) $\log_9 \frac{y}{729}$

$$\log_9 y - \log_9 729$$

$$\log_9 y - \log_9 9^3$$

$$\boxed{\log_9 y - 3}$$

8) $\log_2 4x$

$$\log_2 4 + \log_2 x$$

$$\boxed{2 + \log_2 x}$$

9) $\log_9 \left(\frac{3x^2}{y^5}\right)$

$$\log_9 3x^2 - \log_9 y^5$$

$$\log_9 3 + \log_9 x^2 - 5 \log_9 y$$

$$\boxed{\frac{1}{2} + 2 \log_9 x - 5 \log_9 y}$$

CONDENSE**Condense and simplify the expression.**

10) $\log 2500 + \log 4$

$$\begin{aligned} \log 10000 \\ \log 10^4 \\ 4 \log 10 \\ \boxed{4} \end{aligned}$$

11) $\log_4 48 - \log_4 3$

$$\begin{aligned} \log_4 \frac{48}{3} \\ \log_4 16 \\ \boxed{2} \end{aligned}$$

12) $\log_5 125^8$

$$\begin{aligned} 8 \cdot \log_5 125 \\ 8 \log_5 5^3 \\ 8 \cdot 3 \log_5 5 \\ \boxed{24} \end{aligned}$$

13) $\log_3 2 + \log_3 (4x)$

$$\begin{aligned} \log_3 2 \cdot 4x \\ \boxed{\log_3 8x} \end{aligned}$$

14) $\log_5 100 - \log_5 20$

$$\begin{aligned} \log_5 \frac{100}{20} \\ \log_5 5 \\ \boxed{1} \end{aligned}$$

15) $4 \log_{16} x - 2 \log_{16} 3$

$$\begin{aligned} \log_{16} x^4 - \log_{16} 3^2 \\ \boxed{\log_{16} \frac{x^4}{9}} \end{aligned}$$

16) $4 \log_3 x - 2 \log_3 (4x)$

$$\begin{aligned} \log_3 x^4 - \log_3 (4x)^2 \\ \log_3 \frac{x^4}{16x^2} \\ \boxed{\log_3 \frac{x^2}{16}} \end{aligned}$$

17) $2 \log_4 5 + \frac{1}{2} \log_4 9$

$$\begin{aligned} \log_4 5^2 + \log_4 9^{\frac{1}{2}} \\ \log_4 25 + \log_4 3 \\ \boxed{\log_4 75} \end{aligned}$$

18) $9 \log_4 y + 3 \log_4 (2y)$

$$\begin{aligned} \log_4 y^9 + \log_4 (2y)^3 \\ \log_4 y^9 + \log_4 8y^3 \\ \boxed{\log_4 8y^{12}} \end{aligned}$$

Extension: Using your logarithm properties in order to solve the following log equations.

19) $\log_4 3 + \log_4 x = 2$

$$\begin{aligned} \log_4 3x = 2 \\ 4^2 = 3x \\ 16 = 3x \\ \boxed{x = \frac{16}{3}} \end{aligned}$$

20) $\log_5 x - 2 \log_5 2 = 2$

$$\begin{aligned} \log_5 x - \log_5 4 = 2 \\ \log_5 \frac{x}{4} = 2 \\ 5^2 = \frac{x}{4} \\ 25 = \frac{x}{4} \\ \boxed{x = 100} \end{aligned}$$

21) $2 \log_6 (2x) - \log_6 x = 2$

$$\begin{aligned} \log_6 (2x)^2 - \log_6 x = 2 \\ \log_6 \frac{4x^2}{x} = 2 \\ \log_6 4x = 2 \\ 6^2 = 4x \\ 36 = 4x \\ \boxed{x = 9} \end{aligned}$$