

6.10 NOTES - SOLVING LOG EQUATIONS

OBJECTIVES:

- 1) Use properties of logarithms to evaluate logs, condense logs, and solve log equations.

SOLVING LOG EQUATIONS:

There are three types of logarithmic equations that you will be required to solve.

- 1) $\log_b x = \log_b y$ Both sides of the equation have the same log base.
- 2) $\log = \#$
- 3) $\log_b x = \log_a y$ Each side of the equation has a log with different bases, but one side can be evaluated.

TYPE #1: LOG=LOG

1) $\log_6 x = \log_6 3$

$$\boxed{x = 3}$$

2) $\log_3 x - 4\log_3 2 = \log_3 4$

$$\log_3 \frac{x}{2^4} = \log_3 4$$

$$\frac{x}{16} = 4$$

$$\boxed{x = 64}$$

3) $\log(x+4) + \log(x+6) = \log 3$

$$\log((x+4)(x+6)) = \log 3$$

$$(x+4)(x+6) = 3$$

$$x^2 + 10x + 24 = 3$$

$$x^2 + 10x + 21 = 0$$

$$(x+7)(x+3) = 0$$

$$x = \cancel{-7} \quad \boxed{x = -3}$$

TYPE #2: LOG=#

4) $\log_4 x + \log_4 3 = 2$

$$\log_4 3x = 2$$

$$4^2 = 3x$$

$$16 = 3x$$

$$\boxed{x = \frac{16}{3}}$$

5) $3\log_4 2 + 2\log_4 x = 2$

$$\log_4 2^3 \cdot x^2 = 2$$

$$\log_4 8x^2 = 2$$

$$4^2 = 8x^2$$

$$16 = 8x^2$$

$$x^2 = 2$$

$$\boxed{x = \sqrt{2}} \quad x = \cancel{-\sqrt{2}}$$

6) $2\log_5(-x) - \frac{3}{2}\log_5 25 = -5$

$$\log_5(-x)^2 - \frac{3}{2} \cdot 2 = -5$$

$$\log_5(-x)^2 - 3 = -5$$

$$\log_5 x^2 = -2$$

$$x^2 = 5^{-2}$$

$$x^2 = \frac{1}{25}$$

$$x = \pm \frac{1}{5}$$

$$x = \cancel{\frac{1}{5}} \quad \boxed{x = -\frac{1}{5}}$$

TYPE #3: $\log_b x = \log_a y$

→ you should simplify/evaluate when you can

7) $\log_3 x = 2\log_5 25$

$$\log_3 x = \log_5 (5^2)^2$$

$$\log_3 x = \log_5 5^4$$

simplify!

$$\log_3 x = 4$$

$$3^4 = x$$

$$x = 81$$

8) $\log_3 x - 2\log_3 2 = \log_4 \frac{1}{16}$

$$\log_3 \frac{x}{2^2} = \log_4 4^{-2}$$

$$\log_3 \frac{x}{4} = -2$$

$$3^{-2} = \frac{x}{4}$$

$$\frac{1}{9} = \frac{x}{4}$$

$$\frac{4}{9} = x$$

9) $\log_6 (2x)^2 + \frac{1}{2}\log_6 81 = \log_3 27$

$$\log_6 (2x)^2 \cdot (3^4)^{\frac{1}{2}} = \log_3 3^3$$

$$\log_6 4x^2 \cdot 9 = 3$$

$$6^3 = 36x^2$$

$$\frac{6^3}{6^2} = x^2$$

$$6 = x^2$$

$$x = \sqrt{6} \quad x = -\sqrt{6}$$