MATRICES WITHOUT A CALCULATOR

DEFINING A MATRIX

I. Organizing Data in a Matrix

A *matrix* is a way of organizing data. (Plural is matrices.) For example, the following matrix will be used to organize hair color by gender

$$\begin{array}{ll} \begin{array}{c}\text{Boys}\\ \text{Girls}\end{array} & \begin{bmatrix} \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \end{bmatrix} \\ & \begin{matrix} \text{Black} & \text{Brown} & \text{Red} & \text{Blonde} \end{matrix} \end{array}$$

The size of a matrix, called its **dimensions,** is reported by writing: # of rows × # of columns

The dimension of the matrix above is: $\underline{2} \times \underline{4}$

Give the dimension of each matrix:

1.) $\begin{bmatrix} -12 & 1 \\ 2 & 3 \\ 1 & -5 \end{bmatrix}$ 3×2

2.) $\begin{bmatrix} 1 & 4 & 3 \\ -2 & -8 & -6 \end{bmatrix}$ 2×3

3.) $\begin{bmatrix} 11 \\ 4 \\ 21 \end{bmatrix}$ 3×1

ADDING/SUBTRACTING MATRICES

Note: The dimensions must be the same to be added or subtracted.

1.) $\begin{bmatrix} 2 & 1 \\ 3 & 6 \\ 2 & -3 \end{bmatrix} + \begin{bmatrix} -1 & -9 \\ 4 & 10 \\ 7 & -4 \end{bmatrix}$

$3 \times 2 + 3 \times 2$

$\begin{bmatrix} 1 & -8 \\ 7 & 16 \\ 9 & -7 \end{bmatrix}$

2.) $\begin{bmatrix} 2 & -10 & 15 \end{bmatrix} + \begin{bmatrix} 1 \\ -16 \\ -13 \end{bmatrix}$

$1 \times 3 \quad 3 \times 1$

Can't be added

3.) $\begin{bmatrix} 1 & -2 & 5 \\ 4 & 3 & -6 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 0 \\ 1 & -3 & 7 \end{bmatrix}$

$2 \times 3 + 2 \times 3$ ✓

$\begin{bmatrix} 3 & -1 & 5 \\ 5 & 0 & 1 \end{bmatrix}$

SCALAR MULTIPLICATION

A matrix can be multiplied by a scalar (a number) by multiplying each element of the matrix that scalar:

$$-3\begin{bmatrix} 2 & -23 & 1 \\ 5 & 11 & -8 \end{bmatrix} = \begin{bmatrix} -6 & 69 & -3 \\ -15 & -33 & 24 \end{bmatrix}$$

DETERMINANTS AND INVERSES

The **DETERMINANT** determines whether a matrix has a multiplicative inverse.

We will only take a look at 2 × 2 matrices since matrices that are larger than that are way beyond this course!

The way we take a determinant of a 2 × 2 matrixes is shown below:

$$\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$$

****Fast Fact: If the product equals zero, then the matrix has no multiplicative inverse!**

Find the determinant and state if a multiplicative inverse exists.

1. $\begin{bmatrix} 6 & -3 \\ 9 & 3 \end{bmatrix}$

$\det = (6\cdot 3) - (-3\cdot 9)$
$= 18 - (-27)$
$= 45$

Has an inverse

2. $\begin{bmatrix} 4 & 12 \\ 2 & 6 \end{bmatrix}$

$\det = (6\cdot 4) - (2\cdot 12)$
$= 24 - 24$
$= 0$

No inverse!

3. $\begin{bmatrix} -15 & 1 \\ 4 & 3 \end{bmatrix}$

$\det = (-15\cdot 3) - (1\cdot 4)$
$= -45 - 4$
$= -49$

Has an inverse!

MULTIPLICATION OF TWO MATRICES

A. Determine whether two matrices can be multiplied.
*First write down the dimensions of each matrix.
*If the middle numbers are the same, then the two matrices can be multiplied.
*The outside numbers will result in the dimensions of the product.

Example: $\begin{bmatrix} 2 & -3 & 1 \\ 5 & 10 & -8 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ 7 & 3 \\ 2 & -4 \end{bmatrix}$

B. Multiplying matrices
*Use the idea of taking rows with columns together. You will find the product of each corresponding row & column entry and then take the sum of those entries.

Examples: Determine whether the matrices can be multiplied and then compute the product.

1.) $\begin{bmatrix} 5 & -7 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} -12 & 3 \\ 2 & 0 \end{bmatrix}$ $2\times2 \cdot 2\times2$

can be mult.
2×2 resulting matrix

$$\begin{bmatrix} 5(-12)+-7(2) & 5(3)+-7(0) \\ 1(-12)+4(2) & 1(3)+4(0) \end{bmatrix} = \begin{bmatrix} -60-14 & 15+0 \\ -12+8 & 3+0 \end{bmatrix}$$

$$\begin{bmatrix} -74 & 15 \\ -4 & 3 \end{bmatrix}$$

2.) $\begin{bmatrix} 1 & -3 \\ -2 & 5 \\ -3 & 7 \end{bmatrix}\begin{bmatrix} 2 & -4 \\ 10 & -11 \end{bmatrix}$ $\underbrace{3\times\underbrace{2 \cdot 2}\times2}$

can be mult. 3×2 matrix results

$$\begin{bmatrix} 1\cdot2+-3\cdot10 & 1(-4)+-3(-11) \\ -2(2)+5(10) & -2(-4)+5(-11) \\ -3(2)+7(10) & -3(-4)+7(-11) \end{bmatrix} = \begin{bmatrix} 2-30 & -4+33 \\ -4+50 & 8-55 \\ -6+70 & 12-77 \end{bmatrix}$$

$$\begin{bmatrix} -28 & 29 \\ 46 & -47 \\ 64 & -65 \end{bmatrix}$$

3.) $\begin{bmatrix} 3 \\ 8 \\ -13 \end{bmatrix}\begin{bmatrix} 1 & 9 \end{bmatrix}$ $\underbrace{3\times\underbrace{1 \cdot 1}\times2}_{3\times2}$

$$\begin{bmatrix} 3(1) & 3(9) \\ 8(1) & 8(9) \\ -13(1) & -13(9) \end{bmatrix}$$

$$\begin{bmatrix} 3 & 27 \\ 8 & 72 \\ -13 & -117 \end{bmatrix}$$

4.) $\begin{bmatrix} 1 \\ -2 \\ 6 \\ 7 \end{bmatrix}\begin{bmatrix} 2 & -8 & 7 & -12 \end{bmatrix}$ $\underbrace{4\times\underbrace{1 \cdot 1}\times4}_{4\times4}$

$$\begin{bmatrix} 1\cdot2 & 1(-8) & 1(7) & 1(-12) \\ -2(2) & -2(-8) & -2(7) & -2(-12) \\ 6(2) & 6(-8) & 6(7) & 6(-12) \\ 7(2) & 7(-8) & 7(7) & 7(-12) \end{bmatrix}$$

$$\begin{bmatrix} 2 & -8 & 7 & -12 \\ -4 & 16 & -14 & 24 \\ 12 & -48 & 42 & -72 \\ 14 & -56 & 49 & -84 \end{bmatrix}$$