

9.4 – EQUATIONS OF HYPERBOLAS

OBJECTIVE:

- 1) Complete the square to determine the quadratic shape.
- 2) Write the equation of a hyperbola from given information.

Determine the shape of the quadratic and graph the figure. Include all important information.

1. $x^2 + 4x - 4y^2 + 8y = 36$

$$1(x^2 + 4x + 4) - 4(y^2 - 2y + 1) = 36 + 4 - 4$$

$$(x+2)^2 - 4(y-1)^2 = 36$$

$$\frac{(x+2)^2}{36} - \frac{(y-1)^2}{9} = 1$$

Center: $(-2, 1)$

Vertices: $(4, 1)$ $(-8, 1)$

Foci: $(-2 + 3\sqrt{5}, 1)$ $(-2 - 3\sqrt{5}, 1)$

$$F^2 = r_x^2 + r_y^2$$

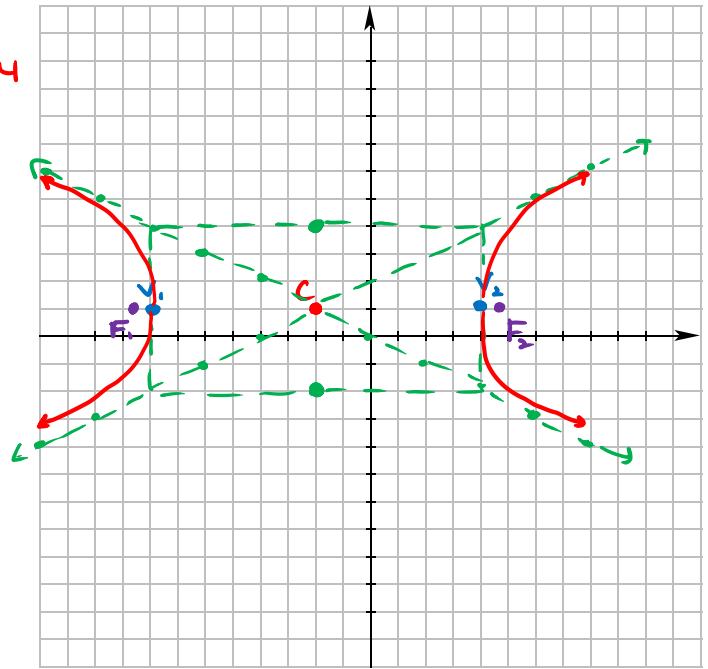
$$F^2 = 36 + 9 \approx \text{between } 6/7 \text{ units}$$

$$F = \sqrt{45} = 3\sqrt{5}$$

Asymptotes: $m = \pm \frac{r_y}{r_x} = \pm \frac{3}{6} = \pm \frac{1}{2}$

Equation:

$$y - 1 = \pm \frac{1}{2}(x + 2)$$



2. $y^2 + 6y - 4x^2 - 8x = 59$

$$-4(x^2 + 2x + 1) + 1(y^2 + 6y + 9) = 59 - 4 + 9$$

$$-4(x+1)^2 + (y+3)^2 = 64$$

$$\frac{-(x+1)^2}{16} + \frac{(y+3)^2}{64} = 1$$

Center: $(-1, -3)$

Vertices: $(-1, -11)$ $(-1, 5)$

Foci: $(-1, -3 + 4\sqrt{5})$ $(-1, -3 - 4\sqrt{5})$

$$F^2 = r_x^2 + r_y^2$$

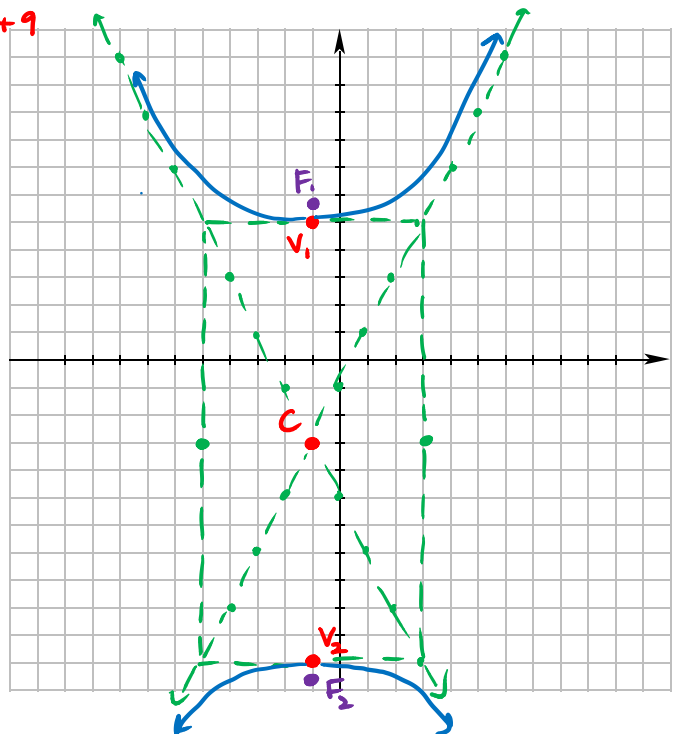
$$= 16 + 64$$

$$F = \sqrt{80} = 4\sqrt{5}$$

between 8 & 9

Asymptotes:

$$m = \pm \frac{r_y}{r_x} = \pm \frac{8}{4} = \pm 2 \quad y + 3 = \pm 2(x + 1)$$



Write the equation of the hyperbola that fits the given information below:

5) Vertices (1, 1) and (1, -3) and Foci $(1, -1 \pm \sqrt{5})$

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6) Vertices (-3, -3) and (5, -3) and Foci (-5, -3) and (7, -3).



Center: (1, -3)

From vertices:

$$r_x = 4$$

From foci: $F = 6$

$$F^2 = r_x^2 + r_y^2$$

$$36 = 16 + r_y^2 \quad r_y^2 = 20$$

$$\frac{(x-1)^2}{16} - \frac{(y+3)^2}{20} = 1$$

7) Vertices (-7, 0) and (7, 0) and conjugate axis of length 10 units.

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