480/81
Day 1
Notes

NATURAL LOGS – NOTES

KEY

OBJECTIVES:

1) Use properties of logarithms to evaluate, condense, and solve equations using natural logs.

The history of mathematics is marked by the discovery of special numbers such as counting numbers, zero, negative numbers, π, and imaginary numbers. In this lesson you will study on e of the most famous numbers of modern times. Like π and i, the number e, is denoted by a letter. The number is called the natural base e, or the Euler number after its discoverer, Leonhard Euler (1707 – 1783).

NATURAL LOGARITHM:

The logarithm with base e is called the natural logarithm, so we can use $\log_e$, but it is more often denoted by ln.

NATURAL LOG: $\log_e x = \ln x$

PROPERTIES OF LOGARITHMS

$\log 10 =$ 1 $\qquad \log 1 =$ 0

$\log_x ab = \log_x a + \log_x b$

$\log_x \frac{a}{b} = \log_x a - \log_x b$

$\log_x a^b = b\log_x a$

LOG PROPERTIES REVISITED, FOR NATURAL LOGS:

1. $\ln e =$ 1

2. $\ln 1 =$ 0

3. $\ln(xy) =$ $\ln x + \ln y$

4. $\ln\left(\frac{x}{y}\right) =$ $\ln x - \ln y$

5. $\ln x^n =$ $n \ln x$

Evaluating Natural Logs!

1. $\ln e$

1

2. $\ln 0$

$\emptyset$

3. $\ln e^5$

By definition: $e^? = e^5$ $\boxed{5}$

By property: $5 \ln e$, $5 \cdot 1$ $\boxed{5}$

4. $\ln e^{-3} + \ln e^{12}$

$-3 + 12$

$\boxed{9}$

5. $\ln\left(\frac{1}{e^{24}}\right)$

$\ln e^{-24}$ $\boxed{-24}$

OR $\ln 1 - \ln e^{24}$, $0 - 24$ $\boxed{-24}$

6. $\frac{(\ln e^2)^6}{(\ln e^3)^4}$

$\frac{(2)^6}{(3)^4}$

$\boxed{\frac{64}{81}}$

Condensing the following expressions.

7. $\ln 16 - \ln 4$

$\ln \frac{16}{4}$

$\boxed{\ln 4}$

8. $3(\ln 3 - \ln x) + (\ln x - \ln 9)$

$3\left(\ln \frac{3}{x}\right) + \ln \frac{x}{9}$

$\ln \left(\frac{3}{x}\right)^3 + \ln \frac{x}{9}$

$\ln \frac{27}{x^3} \cdot \frac{x}{9}$

$\boxed{\ln \frac{3}{x^2}}$

9. $\ln 20 + 2\ln \frac{1}{2} + \ln x$

$\ln 20 + \ln\left(\frac{1}{2}\right)^2 + \ln x$

$\ln 20 + \ln \frac{1}{4} + \ln x$

$\ln 20 \cdot \frac{1}{4} \cdot x$

$\boxed{\ln 5x}$

Solve the natural log equation:

10. $1 - 2\ln x = -4$

$-2\ln x = -5$

$\ln x = \frac{5}{2}$

$\boxed{e^{5/2} = x}$

11. $3\ln(x + 2) = 24$

$\ln(x+2) = -8$

$e^{-8} = x + 2$

$\boxed{x = -2 + \frac{1}{e^8}}$

APPLICATIONS OF NATURAL LOGS:

12. The atmospheric pressure P decreases exponentially with the height above sea level. The equation relating the pressure P and the height H is $P = 14.7e^{-kh}$, where k is a positive constant.

a.) Find k if the pressure is 11.9 pounds per square inch at an altitude of 5000 feet.

$P = 11.9 \quad h = 5{,}000$

$11.9 = 14.7e^{-5000k}$

$\frac{11.9}{14.7} = e^{-5000k}$

$\ln\left(\frac{11.9}{14.7}\right) = \ln\left(e^{-5000k}\right)$

$\ln \frac{11.9}{14.7} = -5000k$

$\boxed{k = \frac{\ln \frac{11.9}{14.7}}{-5000}} \rightarrow k \approx 4.226 \times 10^{-5}$

b.) Find the atmospheric pressure outside an airplane that is flying at an altitude of 22,000 feet.

$P = 14.7e^{-k \cdot 22000}$

(Store "k" in your calculator)

$\boxed{P = 37.248}$

13. The mass of a radioactive substance decreases exponentially with time according to the equation $M = M_0e^{-kt}$, where M_0 is the amount of the radioactive substance at $t=0$ and k is a positive constant that depends on the type of radioactive substance. Assume you start with 20 mg of radioactive substance.

a. Find k for carbon-14 if the half-life of carbon-14 is 5600 years.
NOTE: The half life is the time it takes for HALF of the mass, M_o, to decay.

$M = M_0 e^{-kt}$ $\quad t = 5600 \quad M = .5 M_0$

$.5 = \frac{M}{M_0}$

$\frac{1}{2} = e^{-k \cdot 5600}$

$\ln \frac{1}{2} = -5600k$

$\boxed{\frac{\ln \frac{1}{2}}{-5600} = k}$

b. Find k if the initial amount of radioactive substance was 30 mg.

$M_0 = 30 \quad t = 5660$

$15 = 30e^{-k \cdot 5600}$

$\frac{1}{2} = e^{-5600k}$

$\ln \frac{1}{2} = -5600k$

Same!

$k = \boxed{\frac{\ln \frac{1}{2}}{-5600}}$

c. A bone is found with 30% of its carbon-14 remaining. How old is the bone?

$\frac{3}{10} = e^{-kt}$

$\frac{3}{10} = e^{-\left(\frac{\ln \frac{1}{2}}{-5600}\right)t}$

$\ln \frac{3}{10} = -\left(\frac{\ln \frac{1}{2}}{-5600}\right)t$

$t = \frac{\ln \frac{3}{10}}{\left(\frac{\ln \frac{1}{2}}{5600}\right)}$

$\boxed{\text{9,727 years old}}$