480/81
Day 9
Notes

8.5 NOTES – VARIATION FUNCTIONS

OBJECTIVE:

1) Given a real world situation in which one variable is proportional to a non-integer power of another variable, determine the proportionality constant, and use the resulting variation function as a mathematical model.

EXAMPLES:

1. Ship Power: When a ship is traveling at high speed through the water, most of the power generated by the engines goes into formation of the wake (the waves that trail out behind the ship). At these speeds, the speed of the ship is proportional to the seventh root of the power being generated by the engines. Suppose that you are on a ship going 30 knots (30 nautical miles per hour) and the engines are generating 45,000 horsepower.

a. Write the particular equation expressing speed in terms of power.

P = horse power of engines (P, S)

S = speed of boat (45,000, 30)

$S = K\sqrt[7]{P}$

$30 = K\sqrt[7]{45{,}000}$

$K = \frac{30}{\sqrt[7]{45000}}$ $K \approx 6.49$ STO K!

$S = 6.49\sqrt[7]{P}$

b. The engines are capable of producing 90,000 horsepower. How fast would you expect the ship to go if the Captain gives the order, "Full speed ahead!"?

$P = 90{,}000$

$S = 6.49\sqrt[7]{90{,}000}$

$S \approx 33.1$

33.1 knots

c. Does doubling the power cause the speed to double?

NO!

Doubling the power increases the speed by a factor of $2^{1/7}$

2. From measurements on many rivers, geographers find that the length of a river that drains a particular "basin" of land is approximately proportional to the .6 power of the area of the basin. The Rio Grande is 3034 kilometers long, and drains a basin of about 500,000 square kilometers.

a.) Write the particular equation expressing river length in terms of basin area.

A = area of basin l = length of river

$(500{,}000, 3034)$

$l = k(l)^{.6} = k\,l^{3/5} = k\sqrt[5]{l^3}$

$3034 = k(500{,}000)^{3/5}$

$k = \dfrac{3034}{(500{,}000)^{3/5}}$ $k \approx 1.155$

$\boxed{l = 1.155\,A^{3/5}}$

or

$\boxed{l = 1.155\sqrt[5]{A^3}}$

b.) The Suwannee River flows from the Okefenokee Swamp to the Gulf of Mexico. It drains an area of about 15,000 square kilometers. According to your model, how long is the Suwannee River?

$A = 15{,}000$

STO!

$l = kA^{3/5}$

$l = k(15{,}000)^{.6}$

$\boxed{l \approx 370.01 \text{ km long}}$

c.) The longest river in the world is the 6,700-kilometer Nile. Approximately what area of land does the Nile drain?

$l = 6{,}700$ km

$6700 = k(A)^{3/5}$

$\dfrac{6700}{k} = (A)^{3/5}$

$\left(\dfrac{6700}{k}\right)^{5/3} = \left(A^{3/5}\right)^{5/3}$

$A = \left(\dfrac{6700}{k}\right)^{5/3}$ $\boxed{A \approx 1{,}872{,}414.726 \text{ km}^2}$