13.5
Notes

CONJUGATE ROOTS AND DESCARTES' RULE OF SIGNS

OBJECTIVES: 1) Use the conjugate roots theorem to solve an equation.
2) Determine all of the possible combinations of roots of a polynomial.

THE CONJUGATE ROOTS THEOREM:

Let f(x) be a polynomial, all of whose coefficients are real numbers. Suppose that $a+bi$ is a root of the equation f(x)=0. Then $a-bi$ is also a root. (Complex roots come in pairs!)

THE IRRATIONAL ROOTS THEOREM:

Let f(x) be a polynomial, all of whose coefficients are real numbers. Suppose that $a+b\sqrt{c}$ is a root of the equation f(x)=0. Then $a-b\sqrt{c}$ is also a root.

1) Solve $3x^4-x^3-7x^2+49x-60=0$ if $1+2i$ is a root.

$r_1=1+2i \quad r_2=1-2i$

$(x-(1+2i))(x-(1-2i))$ are factors → or multiply these factors out

or

$r_1 \cdot r_2=c \qquad -(r_1+r_2)=b$

$(1+2i)(1-2i)=c \qquad -(1+2i+1-2i)=b$

$c=1-4i^2 \qquad -(2)=b$

$\boxed{c=5} \qquad \boxed{b=-2}$

x^2-2x+5 is a factor of $3x^4-x^3-7x^2+49x-60$.

Now use long division!

$$\begin{array}{r|l} & \boxed{3x^2+5x-12} \\ \hline x^2-2x+5 & 3x^4-x^3-7x^2+49x-60 \end{array}$$

OR use synthetic division:

$$\begin{array}{r|rrrrr} 1+2i & 3 & -1 & -7 & 49 & -60 \\ & & 3+6i & 10i-10 & -37-24i & 60 \\ \hline & 3 & 2+6i & 10i-17 & 12-24i & \boxed{0} \end{array}$$

$$\begin{array}{r|rrrr} 1-2i & 3 & 2+6i & 10i-17 & 12-24i \\ & & 3-6i & -10i+5 & -12+24i \\ \hline & 3 & 5 & -12 & \boxed{0} \end{array}$$

$\therefore 3x^2+5x-12$ is a factor

$3x^2+5x-12=0$

$(3x-4)(x+3)=0$

$x=\frac{4}{3} \quad x=-3$

Solutions:
$1\pm 2i, \frac{4}{3}, -3$

2) Find a quadratic equation with rational coefficients and a leading coefficient of 1 such that one of the roots is $r_1=2+5\sqrt{3}$.

$c=r_1 \cdot r_2=(2+5\sqrt{3})(2-5\sqrt{3})$

$c=4-25\cdot 3=\boxed{-71}$

$\boxed{x^2-4x-71=0}$

$b=-(2+5\sqrt{3}+2-5\sqrt{3})$

$b=\boxed{-4}$

DESCARTES' RULE OF SIGNS:

For any polynomial f(x), use the number of sign changes in f(x) and f(–x) to predict the maximum number of positive or negative (real) roots.

List all of the possible combinations of roots for the following polynomials:

3) $f(x) = x^3 + x^2 - x + 1$

$f(x) = x^3 + x^2 - x + 1$

2 sign changes: max 2 pos. roots

$f(-x) = -x^3 + x^2 + x + 1$

1 sign change: max 1 neg. root

pos.	neg.	nonreal complex
2	1	0
0	1	2

4) $f(x) = x^3 + x^2 + x + 1$

$f(x) = x^3 + x^2 + x + 1$

0 sign changes: 0 pos. roots

$f(-x) = -x^3 + x^2 - x + 1$

3 sign changes: max 3 neg. roots

pos.	neg.	nonreal
0	3	0
0	1	2

5) $f(x) = x^3 - 14x^2 + 62x - 88$

$f(x) = x^3 - 14x^2 + 62x - 88$

3 sign changes: max 3 pos. roots

$f(-x) = -x^3 - 14x^2 - 62x - 88$

0 sign changes: no neg. roots

pos	neg	non-real
3	0	0
1	0	2

6) $f(x) = x^5 - x^4 - 3x^3 + 2x^2 + x - 5$

$f(x) = x^5 - x^4 - 3x^3 + 2x^2 + x - 5$

3 sign changes: max 3 pos. roots

$f(-x) = -x^5 - x^4 + 3x^3 + 2x^2 - x - 5$

2 sign changes: max 2 neg roots

+	–	i
3	2	0
3	0	2
1	2	2
1	0	4
0	1	4

wrong! not an option!

7) $f(x) = 5x^4 + x^2 - 3x - 2$

$f(x) = 5x^4 + x^2 - 3x - 2$

2 sign changes ⇒ max 2 pos. roots

$f(-x) = 5x^4 + x^2 + 3x - 2$

1 sign change ⇒ max 1 neg root

pos.	neg	nonreal
2	0	2
0	0	4
1	1	2

wrong!

This is not an option! They only desert in pairs!

8) $f(x) = 3x^5 + 2x^4 - 5x^3 + 4x^2 + 6x + 7$

$f(x) = 3x^5 + 2x^4 - 5x^3 + 4x^2 + 6x + 7$

2 sign changes: max 2 pos. roots

$f(-x) = -3x^5 + 2x^4 + 5x^3 + 4x^2 - 6x + 7$

3 sign changes: max 3 neg. roots

+	–	i
2	3	0
0	3	2
2	1	2
0	1	4