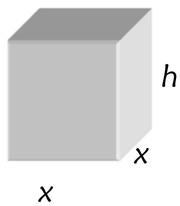
Ch 4
PreCalc

MAX MIN QUIZ REVIEW

Name: KEY

Date: ______________ Period: _______

1. A closed box with a square base is made of material that costs \$10 per m^2 for the top and bottom and \$5 per m^2 for the sides. The volume of the box is $8m^3$. Express the cost of the material for the box as a function of x.



$V = x^2 h$

$8 = x^2 h$

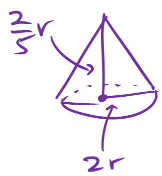
$h = \frac{8}{x^2}$

$SA = 2x^2 + 4xh$

$C(x) = 2x^2(10) + 4x \cdot \frac{8}{x^2}(5)$

$\boxed{C(x) = 20x^2 + \frac{160}{x}}$

2. A pile of sand is in the shape of a cone with a diameter that is 5 times the height. Express the volume of sand as a function of the radius.

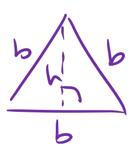


$V = \frac{1}{3}\pi r^2 \cdot h$

$V(r) = \frac{1}{3}\pi r^2 \cdot \frac{2}{5}r$

$\boxed{V(r) = \frac{2}{15}\pi r^3}$

3. Express the area of an equilateral triangle as a function of the height of the triangle.



$A = \frac{1}{2}bh$

$\left(\frac{b}{2}\right)^2 + h^2 = b^2$

$h^2 = b^2 - \frac{b^2}{4}$

$b^2\left(1 - \frac{1}{4}\right) = h^2$

$\frac{3}{4}b^2 = h^2$

$b^2 = \frac{4}{3}h^2$

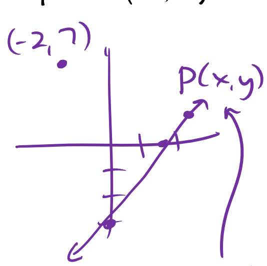
$b = \sqrt{\frac{4}{3}h^2}$

$b = \frac{2}{\sqrt{3}}h$

$A(h) = \frac{1}{2}\left(\frac{2}{\sqrt{3}}h\right)h$

$\boxed{A(h) = \frac{1}{\sqrt{3}}h^2}$

4. $P\,(x, y)$ is an arbitrary point on the line $4x - 2y = 6$. What is the distance from P to the point (-2, 7) as a function of the y coordinate of P?



$x = \frac{1}{2}y + \frac{3}{2}$

$d(y) = \sqrt{\left(\frac{1}{2}y + \frac{3}{2} + 2\right)^2 + (y-7)^2}$

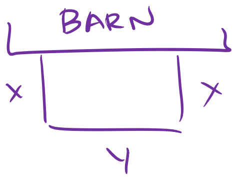
$\boxed{d(y) = \sqrt{\left(\frac{1}{2}y + \frac{7}{2}\right)^2 + (y-7)^2}}$

$4x - 2y = 6$

$4x = 2y + 6$

$x = \frac{1}{2}y + \frac{3}{2}$

5. A rectangular dog pen is constructed using a barn wall as one side and 60m of fencing for the other three sides. Find the dimensions of the pen that give the greatest area.



$2x + y = 60$

$A = xy$

$A(x) = x(-2x+60)$

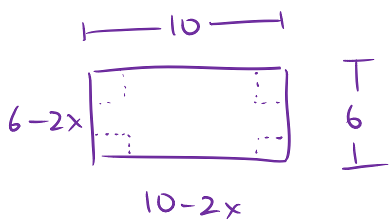
$A(x) = -2x^2 + 60x$

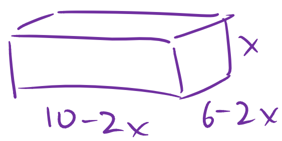
Find where max occurs:

$\frac{-60}{2(-2)} = 15$

$\boxed{\text{Dimensions: } 15 \times 30}$

6. Squares with sides of length x are cut from the corners of a rectangular piece of sheet metal with dimensions of 6 in and 10 in. The metal is then folded to make an open top box. Express the volume as a function of x.





$\boxed{V(x) = x(10-2x)(6-2x)}$

7. A local video store determines that it can rent 10,000 films per month if the rental price is $32 for each film. It also estimates that for each 25-cent reduction in price, 100 more films will be rented. What is the maximum possible income and what rental price per film gives this income?

Income = (Price)(# of units rented)

Find Price Function:

x = # units rented

y = price

$m = \frac{-.25}{100}$ (10,000, 32)

$y - 32 = \frac{-.25}{100}(x - 10{,}000)$

$y = \frac{-.25}{100}x + 57$

$I(x) = \left(\frac{-.25}{100}x + 57\right)x$

$I(x) = \frac{-.25}{100}x^2 + 57x$

Find max:

$x = \frac{-b}{2a} = \frac{-57}{2\left(\frac{-.25}{100}\right)} = 11400$

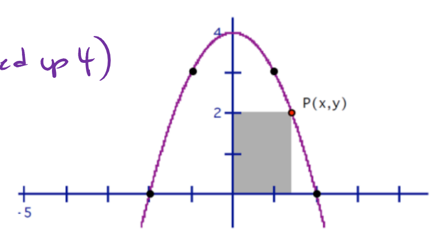
(11400, 324,900) $I = \frac{-.25}{100}(11{,}400)^2 + 57(11{,}400) = \$324{,}900$

Rental Price:

$y = \frac{-.25}{100}(11{,}400) + 57 = \$28.50$

The max income is $324,900 when renting 11,400 films at $28.50 per film.

8. A point P lies on the graph of the quadratic function below. Find the quadratic function and express the area of the rectangle as a function of x.



Quadratic: $y = -x^2 + 4$ (x^2 reflected & translated up 4)

$A = xy$

$A(x) = x(-x^2 + 4)$

$\boxed{A(x) = -x^3 + 4x}$