

## (PART 2) SUM-TO-PRODUCT FORMULAS

**OBJECTIVES:** 1) Use the sum-to-product/product-to-sum formulas to verify identities.

### THE SUM-TO-PRODUCT/PRODUCT-TO-SUM FORMULAS

$$\sin(A + B) + \sin(A - B) = 2\sin A \cos B$$

$$\cos(A + B) + \cos(A - B) = 2\cos A \cos B$$

$$\sin(A + B) - \sin(A - B) = 2\cos A \sin B$$

$$\cos(A + B) - \cos(A - B) = -2\sin A \sin B$$

\*These formulas are proven in Part 8 of your Prove It Notes.

### VERIFYING TRIG IDENTITIES

$$1) \frac{\sin 3x - \sin x}{\cos 3x + \cos x} = \tan x$$

$$\frac{\sin 3x - \sin x}{\cos 3x + \cos x} =$$

$$\frac{2\cancel{\cos 2x} \sin x}{2\cancel{\cos 2x} \cos x} =$$

$$\frac{\sin x}{\cos x} =$$

$$\tan x = \checkmark$$

$$2) \frac{\sin 4x + \sin 2x}{\sin 2x} = \frac{\sin 3x}{\sin x}$$

$$\frac{\cancel{2\sin 3x} \cos x}{\cancel{2\sin x} \cos x} =$$

$$\frac{\sin 3x}{\sin x} = \checkmark$$

$$2) \frac{\cos x - \cos y}{\sin x + \sin y} = -\tan \frac{x-y}{2}$$

$$\frac{-2\sin \frac{x+y}{2} \sin \frac{x-y}{2}}{2\sin \left(\frac{x+y}{2}\right) \cos \left(\frac{x-y}{2}\right)} =$$

$$\frac{-\sin \left(\frac{x-y}{2}\right)}{\cos \left(\frac{x-y}{2}\right)} =$$

$$-\tan \left(\frac{x-y}{2}\right) = \checkmark$$

## VERIFYING TRIG IDENTITIES

$$\begin{aligned} 3) \quad 4 \cos x \cos 2x \sin 3x &= \sin 2x + \sin 4x + \sin 6x \\ &\quad \sin 4x + \sin 2x \\ &= 2 \sin 3x \cos x + \sin 6x \\ &= 2 \sin 3x \cos x + 2 \sin 3x \cos 3x \\ &= 2 \sin 3x (\cos x + \cos 3x) \\ &= 2 \sin 3x (2 \cos 2x \cos x) \\ \checkmark &= 4 \cos x \cos 2x \sin 3x \end{aligned}$$

## REVIEW PROBLEMS

4) Evaluate  $\cos 75^\circ - \cos 15^\circ$ .

Use  $\cos(A+B) - \cos(A-B) = -2 \sin A \sin B$

$$(A+B) + (A-B) = 2A$$

$$75 + 15 = 2A$$

$$A = 45$$

$$(A+B) - (A-B) = 2B$$

$$75 - 15 = 2B$$

$$B = 30$$

$$\cos 75 - \cos 15 = -2 \sin 45 \sin 30$$

$$= -2 \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$= \boxed{\frac{-\sqrt{2}}{2}}$$

5) Evaluate  $\sin 75^\circ + \sin 15^\circ$ .

Use  $\sin(A+B) + \sin(A-B) = 2 \sin A \cos B$

$$(A+B) + (A-B) = 2A$$

$$75 + 15 = 2A$$

$$A = 45$$

$$(A+B) - (A-B) = 2B$$

$$75 - 15 = 2B$$

$$B = 30$$

$$\sin 75 + \sin 15 = 2 \sin 45 \cos 30$$

$$= 2 \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$$

$$= \boxed{\frac{\sqrt{6}}{2}}$$