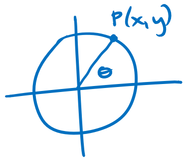
PROVE IT NOTES

I. BASIC DEFINITIONS

7 pts



1) $\cos\theta = x$

2) $\sin\theta = y$

3) $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{y}{x}$

4) $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{x}$

5) $\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{y}$

6) $\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{x}{y}$

II. PYTHAGOREAN IDENTITIES

5 pts

$x^2 + y^2 = 1$

$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

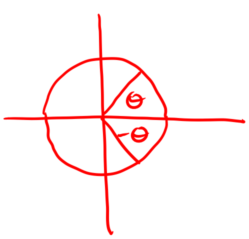
7) $\cos^2\theta + \sin^2\theta = 1$

8) $1 + \tan^2\theta = \sec^2\theta$

9) $\cot^2\theta + 1 = \csc^2\theta$

III. OPPOSITE ANGLES

5 pts



10) $\sin(-\theta) = -\sin\theta$

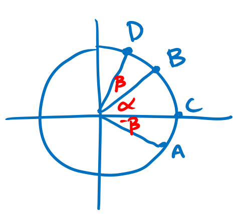
11) $\cos\theta = \cos(-\theta)$

$$\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin\theta}{\cos\theta} = -\tan\theta$$

12) $\tan(-\theta) = -\tan\theta$

IV. ANGLE ADDITION

8 pts



$A(\cos\beta, -\sin\beta)$ $B(\cos\alpha, \sin\alpha)$ $C(1,0)$ $D(\cos(\alpha+\beta), \sin(\alpha+\beta))$

$AB = CD$

$$\sqrt{(\cos\alpha - \cos\beta)^2 + (\sin\alpha + \sin\beta)^2} = \sqrt{(\cos(\alpha+\beta) - 1)^2 + \sin^2(\alpha+\beta)}$$

$$\cos^2\alpha - 2\cos\alpha\cos\beta + \cos^2\beta + \sin^2\alpha + 2\sin\alpha\sin\beta + \sin^2\beta = \cos^2(\alpha+\beta) - 2\cos(\alpha+\beta) + 1 + \sin^2(\alpha+\beta)$$

$$-2\cos\alpha\cos\beta + 2\sin\alpha\sin\beta + 2 = 2 - 2\cos(\alpha+\beta)$$

$$-2\cos\alpha\cos\beta + 2\sin\alpha\sin\beta = -2\cos(\alpha+\beta)$$

13) $\boxed{\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta}$

$\cos(\alpha-\beta) = \cos(\alpha + -\beta) = \cos\alpha\cos(-\beta) - \sin\alpha\sin(-\beta)$

$= \cos\alpha\cos\beta + \sin\alpha\sin\beta$

14) $\boxed{\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta}$

V. COMPLIMENTS

{4 pts}

$$\cos\left(\frac{\pi}{2}-\alpha\right) = \cos\frac{\pi}{2}\cos\alpha + \sin\frac{\pi}{2}\sin\alpha = 0 + \sin\alpha$$

15) $$\boxed{\cos\left(\frac{\pi}{2}-\alpha\right) = \sin\alpha}$$

$$\sin\left(\frac{\pi}{2}-\alpha\right) = \cos\left(\frac{\pi}{2}-\left(\frac{\pi}{2}-\alpha\right)\right) = \cos(\alpha)$$

16) $$\boxed{\sin\left(\frac{\pi}{2}-\alpha\right) = \cos\alpha}$$

IV. ANGLE ADDITION

{9 pts}

$$\sin(\alpha+\beta) = \cos\left(\frac{\pi}{2}-(\alpha+\beta)\right) = \cos\left(\left(\frac{\pi}{2}-\alpha\right)-\beta\right)$$

$$= \cos\left(\frac{\pi}{2}-\alpha\right)\cos\beta + \sin\left(\frac{\pi}{2}-\alpha\right)\sin\beta = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

17) $$\boxed{\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta}$$

$$\sin(\alpha-\beta) = \sin(\alpha+-\beta) = \sin\alpha\cos(-\beta) + \cos\alpha\sin(-\beta)$$

$$= \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

18) $$\boxed{\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta}$$

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta - \sin\alpha\sin\beta}\left(\frac{\frac{1}{\cos\alpha\cos\beta}}{\frac{1}{\cos\alpha\cos\beta}}\right)$$

$$= \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$$

19) $$\boxed{\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}}$$

$$\tan(\alpha-\beta) = \tan(\alpha+-\beta) = \frac{\tan\alpha + \tan(-\beta)}{1-\tan\alpha\tan(-\beta)}$$

$$= \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$$

20) $$\boxed{\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}}$$

VI. DOUBLE ANGLE

{10 pts}

$\sin 2\theta = \sin(\theta+\theta) = \sin\theta\cos\theta + \cos\theta\sin\theta$

$= 2\sin\theta\cos\theta$

21) $\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos(\theta+\theta) = \cos\theta\cos\theta - \sin\theta\sin\theta$

$= \cos^2\theta - \sin^2\theta$

22) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - \sin^2\theta - \sin^2\theta$

$= 1 - 2\sin^2\theta$

23) $\cos 2\theta = 1 - 2\sin^2\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = \cos^2\theta - (1 - \cos^2\theta)$

$= 2\cos^2\theta - 1$

24) $\cos 2\theta = 2\cos^2\theta - 1$

$\tan 2\theta = \dfrac{\tan\theta + \tan\theta}{1 - \tan\theta\tan\theta} = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

25) $\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

VII. HALF ANGLE

{11 pts}

$\cos 2\theta = 1 - 2\sin^2\theta \Rightarrow 2\sin^2\theta = 1 - \cos 2\theta \Rightarrow$

26) $\sin^2\theta = \dfrac{1 - \cos 2\theta}{2}$

let $x = 2\theta$, $\theta = \frac{x}{2}$

$\sin^2\left(\frac{x}{2}\right) = \dfrac{1 - \cos x}{2}$

27) $\sin\frac{x}{2} = \pm\sqrt{\dfrac{1 - \cos x}{2}}$

$\cos 2\theta = 2\cos^2\theta - 1 \Rightarrow \cos 2\theta + 1 = 2\cos^2\theta$

28) $\cos^2\theta = \dfrac{1 + \cos 2\theta}{2}$

let $x = 2\theta$, $\theta = \frac{x}{2}$

$\cos^2\left(\frac{x}{2}\right) = \dfrac{1 + \cos x}{2}$

29) $\cos\frac{x}{2} = \pm\sqrt{\dfrac{1 + \cos x}{2}}$

$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\pm\sqrt{\frac{1-\cos x}{2}}}{\pm\sqrt{\frac{1+\cos x}{2}}} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}} \cdot \sqrt{\frac{1+\cos x}{1+\cos x}}$$

$$= \pm\sqrt{\frac{1-\cos^2 x}{(1+\cos x)^2}} = \frac{\pm\sqrt{\sin^2 x}}{1+\cos x} = \frac{\sin x}{1+\cos x}$$

30) $\tan\frac{x}{2} = \dfrac{\sin x}{1 + \cos x}$

VIII. SUM TO PRODUCT

{9 pts}

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\sin(x-y) = \sin x \cos y - \cos x \sin y$

Add: 31) $\sin(x+y) + \sin(x-y) = 2\sin x \cos y$

Subtract: 32) $\sin(x+y) - \sin(x-y) = 2\cos x \sin y$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

Add: 33) $\cos(x+y) + \cos(x-y) = 2\cos x \cos y$

Subtract: 34) $\cos(x+y) - \cos(x-y) = -2\sin x \sin y$

Note: If $x > y$, then

$(x+y) + (x-y) = 2x$

$(x+y) - (x-y) = 2y$

} Ex. Credit (+3 pts)