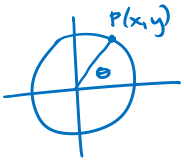


PROVE IT NOTES

I. BASIC DEFINITIONS

7 pts



1) $\cos \theta = x$

2) $\sin \theta = y$

3) $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{y}{x}$

4) $\sec \theta = \frac{1}{\cos \theta} = \frac{1}{x}$

5) $\csc \theta = \frac{1}{\sin \theta} = \frac{1}{y}$

6) $\cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{x}{y}$

II. PYTHAGOREAN IDENTITIES

5 pts

$$x^2 + y^2 = 1$$

$$\frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$$

$$\frac{\cos^2 \theta + \sin^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta}$$

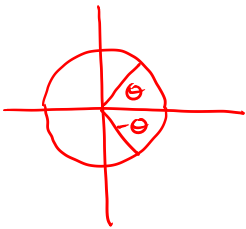
7) $\cos^2 \theta + \sin^2 \theta = 1$

8) $1 + \tan^2 \theta = \sec^2 \theta$

9) $\cot^2 \theta + 1 = \csc^2 \theta$

III. OPPOSITE ANGLES

5 pts



10) $\sin(-\theta) = -\sin \theta$

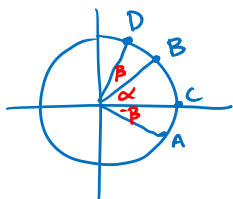
11) $\cos \theta = \cos(-\theta)$

$$\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin \theta}{\cos \theta} = -\tan \theta$$

12) $\tan(-\theta) = -\tan \theta$

IV. ANGLE ADDITION

8 pts



A(cos beta, -sin beta) B(cos alpha, sin alpha) C(1, 0) D(cos(alpha+beta), sin(alpha+beta))

AB = CD

$$\sqrt{(\cos \alpha - \cos \beta)^2 + (\sin \alpha + \sin \beta)^2} = \sqrt{(\cos(\alpha + \beta) - 1)^2 + \sin^2(\alpha + \beta)}$$

$$\cos^2 \alpha - 2 \cos \alpha \cos \beta + \cos^2 \beta + \sin^2 \alpha + 2 \sin \alpha \sin \beta + \sin^2 \beta = \cos^2(\alpha + \beta) - 2 \cos(\alpha + \beta) + 1 + \sin^2(\alpha + \beta)$$

$$-2 \cos \alpha \cos \beta + 2 \sin \alpha \sin \beta + 2 = 2 - 2 \cos(\alpha + \beta)$$

$$-2 \cos \alpha \cos \beta + 2 \sin \alpha \sin \beta = -2 \cos(\alpha + \beta)$$

13) $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

$$\begin{aligned} \cos(\alpha - \beta) &= \cos(\alpha + (-\beta)) = \cos \alpha \cos(-\beta) - \sin \alpha \sin(-\beta) \\ &= \cos \alpha \cos \beta + \sin \alpha \sin \beta \end{aligned}$$

14) $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$

V. COMPLIMENTS

4 pts

$$\cos\left(\frac{\pi}{2}-\alpha\right) = \cos\frac{\pi}{2}\cos\alpha + \sin\frac{\pi}{2}\sin\alpha = 0 + \sin\alpha$$

$$15) \boxed{\cos\left(\frac{\pi}{2}-\alpha\right) = \sin\alpha}$$

$$\sin\left(\frac{\pi}{2}-\alpha\right) = \cos\left(\frac{\pi}{2}-\left(\frac{\pi}{2}-\alpha\right)\right) = \cos(\alpha)$$

$$16) \boxed{\sin\left(\frac{\pi}{2}-\alpha\right) = \cos\alpha}$$

IV. ANGLE ADDITION

9 pts

$$\begin{aligned}\sin(\alpha+\beta) &= \cos\left(\frac{\pi}{2}-(\alpha+\beta)\right) = \cos\left(\frac{\pi}{2}-\alpha\right)\cos\beta + \sin\left(\frac{\pi}{2}-\alpha\right)\sin\beta \\ &= \sin\alpha\cos\beta + \cos\alpha\sin\beta\end{aligned}$$

17)

$$\boxed{\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta}$$

$$\begin{aligned}\sin(\alpha-\beta) &= \sin(\alpha+(-\beta)) = \sin\alpha\cos(-\beta) + \cos\alpha\sin(-\beta) \\ &= \sin\alpha\cos\beta - \cos\alpha\sin\beta\end{aligned}$$

18)

$$\boxed{\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta}$$

$$\begin{aligned}\tan(\alpha+\beta) &= \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta - \sin\alpha\sin\beta} \left(\frac{\frac{1}{\cos\alpha\cos\beta}}{\frac{1}{\cos\alpha\cos\beta}} \right) \\ &= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}\end{aligned}$$

19)

$$\boxed{\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}}$$

$$\begin{aligned}\tan(\alpha-\beta) &= \tan(\alpha+(-\beta)) = \frac{\tan\alpha + \tan(-\beta)}{1 - \tan\alpha\tan(-\beta)} \\ &= \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}\end{aligned}$$

20)

$$\boxed{\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}}$$

VI. DOUBLE ANGLE

10 pts

$$\begin{aligned}\sin 2\theta &= \sin(\theta + \theta) = \sin\theta \cos\theta + \cos\theta \sin\theta \\ &= 2\sin\theta \cos\theta\end{aligned}$$

$$21) \boxed{\sin 2\theta = 2\sin\theta \cos\theta}$$

$$\begin{aligned}\cos 2\theta &= \cos(\theta + \theta) = \cos\theta \cos\theta - \sin\theta \sin\theta \\ &= \cos^2\theta - \sin^2\theta\end{aligned}$$

$$22) \boxed{\cos 2\theta = \cos^2\theta - \sin^2\theta}$$

$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta = 1 - \sin^2\theta - \sin^2\theta \\ &= 1 - 2\sin^2\theta\end{aligned}$$

$$23) \boxed{\cos 2\theta = 1 - 2\sin^2\theta}$$

$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta = \cos^2\theta - (1 - \cos^2\theta) \\ &= 2\cos^2\theta - 1\end{aligned}$$

$$24) \boxed{\cos 2\theta = 2\cos^2\theta - 1}$$

$$\tan 2\theta = \frac{\tan\theta + \tan\theta}{1 - \tan\theta \tan\theta} = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$25) \boxed{\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}}$$

VII. HALF ANGLE

11 pts

$$\cos 2\theta = 1 - 2\sin^2\theta \Rightarrow 2\sin^2\theta = 1 - \cos 2\theta \Rightarrow$$

$$26) \boxed{\sin^2\theta = \frac{1 - \cos 2\theta}{2}} \quad \text{let } x = 2\theta \quad \sin^2\left(\frac{x}{2}\right) = \frac{1 - \cos x}{2}$$

$$27) \boxed{\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}}$$

$$\cos 2\theta = 2\cos^2\theta - 1 \Rightarrow \cos 2\theta + 1 = 2\cos^2\theta$$

$$28) \boxed{\cos^2\theta = \frac{1 + \cos 2\theta}{2}} \quad \text{let } x = 2\theta \quad \cos^2\left(\frac{x}{2}\right) = \frac{1 + \cos x}{2}$$

$$29) \boxed{\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}}$$

$$\begin{aligned}\tan \frac{x}{2} &= \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\pm \sqrt{\frac{1 - \cos x}{2}}}{\pm \sqrt{\frac{1 + \cos x}{2}}} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}} \cdot \sqrt{\frac{1 + \cos x}{1 + \cos x}} \\ &= \pm \sqrt{\frac{1 - \cos^2 x}{(1 + \cos x)^2}} = \pm \frac{\sqrt{\sin^2 x}}{1 + \cos x} = \frac{\sin x}{1 + \cos x}\end{aligned}$$

$$30) \boxed{\tan \frac{x}{2} = \frac{\sin x}{1 + \cos x}}$$

VIII. SUM TO PRODUCT

9 pts

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$

$$\text{Add: } 31) \quad \boxed{\sin(x+y) + \sin(x-y) = 2 \sin x \cos y}$$

$$\text{Subtract: } 32) \quad \boxed{\sin(x+y) - \sin(x-y) = 2 \cos x \sin y}$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\text{Add: } 33) \quad \boxed{\cos(x+y) + \cos(x-y) = 2 \cos x \cos y}$$

$$\text{Subtract: } 34) \quad \boxed{\cos(x+y) - \cos(x-y) = -2 \sin x \sin y}$$

Note: If $x > y$, then

$$(x+y) + (x-y) = 2x$$

$$(x+y) - (x-y) = 2y$$

} Ex. Credit
+3 pts