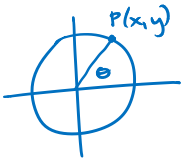


# PROVE IT NOTES

## I. BASIC DEFINITIONS

7 pts



1)  $\cos \theta = x$

2)  $\sin \theta = y$

3)  $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{y}{x}$

4)  $\sec \theta = \frac{1}{\cos \theta} = \frac{1}{x}$

5)  $\csc \theta = \frac{1}{\sin \theta} = \frac{1}{y}$

6)  $\cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{x}{y}$

## II. PYTHAGOREAN IDENTITIES

5 pts

$x^2 + y^2 = 1$

$\frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$

$\frac{\cos^2 \theta + \sin^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta}$

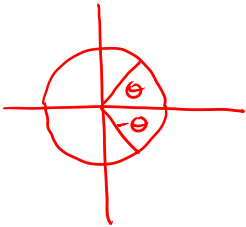
7)  $\cos^2 \theta + \sin^2 \theta = 1$

8)  $1 + \tan^2 \theta = \sec^2 \theta$

9)  $\cot^2 \theta + 1 = \csc^2 \theta$

## III. OPPOSITE ANGLES

5 pts



10)  $\sin(-\theta) = -\sin \theta$

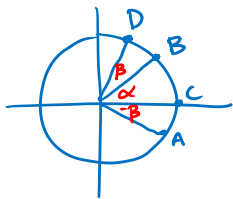
11)  $\cos \theta = \cos(-\theta)$

$\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin \theta}{\cos \theta} = -\tan \theta$

12)  $\tan(-\theta) = -\tan \theta$

## IV. ANGLE ADDITION

8 pts



A(cos beta, -sin beta) B(cos alpha, sin alpha) C(1, 0) D(cos(alpha+beta), sin(alpha+beta))

AB = CD

$\sqrt{(\cos \alpha - \cos \beta)^2 + (\sin \alpha + \sin \beta)^2} = \sqrt{(\cos(\alpha + \beta) - 1)^2 + \sin^2(\alpha + \beta)}$

$\cos^2 \alpha - 2 \cos \alpha \cos \beta + \cos^2 \beta + \sin^2 \alpha + 2 \sin \alpha \sin \beta + \sin^2 \beta = \cos^2(\alpha + \beta) - 2 \cos(\alpha + \beta) + 1 + \sin^2(\alpha + \beta)$

$-2 \cos \alpha \cos \beta + 2 \sin \alpha \sin \beta + 2 = 2 - 2 \cos(\alpha + \beta)$

$-2 \cos \alpha \cos \beta + 2 \sin \alpha \sin \beta = -2 \cos(\alpha + \beta)$

13)  $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

$\cos(\alpha - \beta) = \cos(\alpha + (-\beta)) = \cos \alpha \cos(-\beta) - \sin \alpha \sin(-\beta)$   
 $= \cos \alpha \cos \beta + \sin \alpha \sin \beta$

14)  $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$

## V. COMPLIMENTS

4 pts

$$\cos\left(\frac{\pi}{2}-\alpha\right) = \cos\frac{\pi}{2}\cos\alpha + \sin\frac{\pi}{2}\sin\alpha = 0 + \sin\alpha$$

$$15) \boxed{\cos\left(\frac{\pi}{2}-\alpha\right) = \sin\alpha}$$

$$\sin\left(\frac{\pi}{2}-\alpha\right) = \cos\left(\frac{\pi}{2}-\left(\frac{\pi}{2}-\alpha\right)\right) = \cos(\alpha)$$

$$16) \boxed{\sin\left(\frac{\pi}{2}-\alpha\right) = \cos\alpha}$$

## IV. ANGLE ADDITION

9 pts

$$\begin{aligned}\sin(\alpha+\beta) &= \cos\left(\frac{\pi}{2}-(\alpha+\beta)\right) = \cos\left(\frac{\pi}{2}-\alpha\right)\cos\beta + \sin\left(\frac{\pi}{2}-\alpha\right)\sin\beta \\ &= \sin\alpha\cos\beta + \cos\alpha\sin\beta\end{aligned}$$

17)

$$\boxed{\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta}$$

$$\begin{aligned}\sin(\alpha-\beta) &= \sin(\alpha+(-\beta)) = \sin\alpha\cos(-\beta) + \cos\alpha\sin(-\beta) \\ &= \sin\alpha\cos\beta - \cos\alpha\sin\beta\end{aligned}$$

18)

$$\boxed{\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta}$$

$$\begin{aligned}\tan(\alpha+\beta) &= \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta - \sin\alpha\sin\beta} \left( \frac{\frac{1}{\cos\alpha\cos\beta}}{\frac{1}{\cos\alpha\cos\beta}} \right) \\ &= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}\end{aligned}$$

19)

$$\boxed{\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}}$$

$$\begin{aligned}\tan(\alpha-\beta) &= \tan(\alpha+(-\beta)) = \frac{\tan\alpha + \tan(-\beta)}{1 - \tan\alpha\tan(-\beta)} \\ &= \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}\end{aligned}$$

20)

$$\boxed{\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}}$$