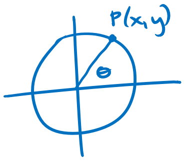
PROVE IT NOTES

I. BASIC DEFINITIONS

7 pts



1) $\cos\theta = x$

2) $\sin\theta = y$

3) $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{y}{x}$

4) $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{x}$

5) $\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{y}$

6) $\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{x}{y}$

II. PYTHAGOREAN IDENTITIES

5 pts

$x^2 + y^2 = 1$

$\dfrac{\cos^2\theta}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta}$

$\dfrac{\cos^2\theta}{\sin^2\theta} + \dfrac{\sin^2\theta}{\sin^2\theta} = \dfrac{1}{\sin^2\theta}$

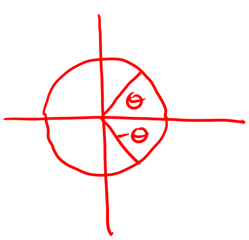
7) $\boxed{\cos^2\theta + \sin^2\theta = 1}$

8) $\boxed{1 + \tan^2\theta = \sec^2\theta}$

9) $\boxed{\cot^2\theta + 1 = \csc^2\theta}$

III. OPPOSITE ANGLES

5 pts



10) $\boxed{\sin(-\theta) = -\sin\theta}$

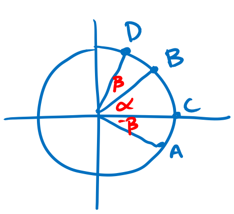
11) $\boxed{\cos\theta = \cos(-\theta)}$

$\tan(-\theta) = \dfrac{\sin(-\theta)}{\cos(-\theta)} = \dfrac{-\sin\theta}{\cos\theta} = -\tan\theta$

12) $\boxed{\tan(-\theta) = -\tan\theta}$

IV. ANGLE ADDITION

8 pts



$A(\cos\beta, -\sin\beta)$ $\quad B(\cos\alpha, \sin\alpha)$ $\quad C(1,0)$ $\quad D(\cos(\alpha+\beta), \sin(\alpha+\beta))$

$AB = CD$

$\sqrt{(\cos\alpha - \cos\beta)^2 + (\sin\alpha + \sin\beta)^2} = \sqrt{(\cos(\alpha+\beta) - 1)^2 + \sin^2(\alpha+\beta)}$

$\cos^2\alpha - 2\cos\alpha\cos\beta + \cos^2\beta + \sin^2\alpha + 2\sin\alpha\sin\beta + \sin^2\beta = \cos^2(\alpha+\beta) - 2\cos(\alpha+\beta) + 1 + \sin^2(\alpha+\beta)$

$-2\cos\alpha\cos\beta + 2\sin\alpha\sin\beta + 2 = 2 - 2\cos(\alpha+\beta)$

$-2\cos\alpha\cos\beta + 2\sin\alpha\sin\beta = -2\cos(\alpha+\beta)$

13) $\boxed{\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta}$

$\cos(\alpha-\beta) = \cos(\alpha + -\beta) = \cos\alpha\cos(-\beta) - \sin\alpha\sin(-\beta)$

$= \cos\alpha\cos\beta + \sin\alpha\sin\beta$

14) $\boxed{\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta}$

V. COMPLIMENTS

4 pts

$$\cos\left(\frac{\pi}{2}-\alpha\right)=\cos\frac{\pi}{2}\cos\alpha+\sin\frac{\pi}{2}\sin\alpha = 0+\sin\alpha$$

15) $\boxed{\cos\left(\frac{\pi}{2}-\alpha\right)=\sin\alpha}$

$$\sin\left(\frac{\pi}{2}-\alpha\right)=\cos\left(\frac{\pi}{2}-\left(\frac{\pi}{2}-\alpha\right)\right)=\cos(\alpha)$$

16) $\boxed{\sin\left(\frac{\pi}{2}-\alpha\right)=\cos\alpha}$

IV. ANGLE ADDITION

9 pts

$$\sin(\alpha+\beta)=\cos\left(\frac{\pi}{2}-(\alpha+\beta)\right)=\cos\left(\left(\frac{\pi}{2}-\alpha\right)-\beta\right)$$

$$=\cos\left(\frac{\pi}{2}-\alpha\right)\cos\beta+\sin\left(\frac{\pi}{2}-\alpha\right)\sin\beta=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$

17) $\boxed{\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta}$

$$\sin(\alpha-\beta)=\sin(\alpha+-\beta)=\sin\alpha\cos(-\beta)+\cos\alpha\sin(-\beta)$$

$$=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$

18) $\boxed{\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta}$

$$\tan(\alpha+\beta)=\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}=\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}\left(\frac{\frac{1}{\cos\alpha\cos\beta}}{\frac{1}{\cos\alpha\cos\beta}}\right)$$

$$=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

19) $\boxed{\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}}$

$$\tan(\alpha-\beta)=\tan(\alpha+-\beta)=\frac{\tan\alpha+\tan(-\beta)}{1-\tan\alpha\tan(-\beta)}$$

$$=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

20) $\boxed{\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}}$