

VI. DOUBLE ANGLE

10 pts

$$\begin{aligned}\sin 2\theta &= \sin(\theta + \theta) = \sin\theta \cos\theta + \cos\theta \sin\theta \\ &= 2\sin\theta \cos\theta\end{aligned}$$

$$21) \quad \boxed{\sin 2\theta = 2\sin\theta \cos\theta}$$

$$\begin{aligned}\cos 2\theta &= \cos(\theta + \theta) = \cos\theta \cos\theta - \sin\theta \sin\theta \\ &= \cos^2\theta - \sin^2\theta\end{aligned}$$

$$22) \quad \boxed{\cos 2\theta = \cos^2\theta - \sin^2\theta}$$

$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta = 1 - \sin^2\theta - \sin^2\theta \\ &= 1 - 2\sin^2\theta\end{aligned}$$

$$23) \quad \boxed{\cos 2\theta = 1 - 2\sin^2\theta}$$

$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta = \cos^2\theta - (1 - \cos^2\theta) \\ &= 2\cos^2\theta - 1\end{aligned}$$

$$24) \quad \boxed{\cos 2\theta = 2\cos^2\theta - 1}$$

$$\tan 2\theta = \frac{\tan\theta + \tan\theta}{1 - \tan\theta \tan\theta} = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$25) \quad \boxed{\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}}$$

VII. HALF ANGLE

11 pts

$$\cos 2\theta = 1 - 2\sin^2\theta \Rightarrow 2\sin^2\theta = 1 - \cos 2\theta \Rightarrow$$

$$26) \quad \boxed{\sin^2\theta = \frac{1 - \cos 2\theta}{2}} \quad \text{let } x = 2\theta \quad \sin^2\left(\frac{x}{2}\right) = \frac{1 - \cos x}{2}$$

$$27) \quad \boxed{\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}}$$

$$\cos 2\theta = 2\cos^2\theta - 1 \Rightarrow \cos 2\theta + 1 = 2\cos^2\theta$$

$$28) \quad \boxed{\cos^2\theta = \frac{1 + \cos 2\theta}{2}} \quad \text{let } x = 2\theta \quad \cos^2\left(\frac{x}{2}\right) = \frac{1 + \cos x}{2}$$

$$29) \quad \boxed{\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}}$$

$$\begin{aligned}\tan \frac{x}{2} &= \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\pm \sqrt{\frac{1 - \cos x}{2}}}{\pm \sqrt{\frac{1 + \cos x}{2}}} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}} \cdot \left(\frac{1 + \cos x}{1 + \cos x}\right) \\ &= \pm \sqrt{\frac{1 - \cos^2 x}{(1 + \cos x)^2}} = \pm \frac{\sqrt{\sin^2 x}}{1 + \cos x} = \frac{\sin x}{1 + \cos x}\end{aligned}$$

$$30) \quad \boxed{\tan \frac{x}{2} = \frac{\sin x}{1 + \cos x}}$$