DIVISION OF POLYNOMIALS

OBJECTIVES: 1) Use long division to find a quotient and remainder.
2) Use synthetic division where applicable.
3) Write an expression of the form $x^n - a^n$ as a product of two factors.

VOCABULARY

Consider 9 divided by 2. The quotient is 4 and the remainder is 1.

$$2\overline{)9}\quad \text{quotient } 4,\ 8,\ 1$$

$$\text{Divisor}\overline{)\text{Dividend}}^{\text{Quotient}} + \text{Remainder}$$

$$\frac{\text{Dividend}}{\text{Divisor}}$$

$$\text{Dividend} = \text{Divisor} \bullet \text{Quotient} + \text{Remainder}$$

LONG DIVISION

1) Let $p(x) = x^5 + 32$ and $d(x) = x - 3$.

Use long division to find the polynomials $q(x)$ and $R(x)$ such that $p(x) = d(x) \bullet q(x) + R(x)$.

$$\frac{x^5+32}{x-3} =$$

$$\boxed{x^4+3x^3+9x^2+27x+81 - \frac{275}{x-3}}$$

Long division: $x-3 \overline{)x^5+0x^4+0x^3+0x^2+0x+32}$, quotient $x^4+3x^3+9x^2+27x+81$

$-(x^5-3x^4)$

$3x^4+0x^3$

$-(3x^4-9x^3)$

$9x^3+0x^2$

$-(9x^3-27x^2)$

$27x^2+0x$

$-(27x^2-81x)$

$81x+32$

$-(81x-243)$

-275 remainder!

Remainder -275

In $p(x)=d(x)\cdot q(x)+R(x)$ form:

$$x^5+32=(x-3)(x^4+3x^3+9x^2+27x+81x)-275$$

2) Let $p(x) = 4x^3 + 4x^2 - x - 3$ and $d(x) = x^2 + 1$.

Use long division to find the polynomials $q(x)$ and $R(x)$ such that $p(x) = d(x) \bullet q(x) + R(x)$.

$$\frac{4x^3+4x^2-x-3}{x^2+1} =$$

Long division: $x^2+0x+1 \overline{)4x^3+4x^2-x-3}$, quotient $4x+4$

$-(4x^3+0x^2+4x)$

$4x^2-5x-3$

$-(4x^2+0x+4)$

$-5x-7$ remainder

$$= 4x+4+\frac{-5x-7}{x^2+1}$$

$$= \boxed{4x+4-\frac{5x+7}{x^2+1}}$$

In $p(x)=d(x)\cdot q(x)+R(x)$ form:

$$\underbrace{4x^3+4x^2-x-3}_{p(x)} = \underbrace{(x^2+1)}_{d(x)}\underbrace{(4x+4)}_{q(x)} - \underbrace{(5x+7)}_{R(x)}$$

SYNTHETIC DIVISION

Works only for divisors of the form $x - r$!!

3) $\dfrac{x^3 - 1}{x - 1}$

$$\begin{array}{c|cccc} 1 & 1 & 0 & 0 & -1 \\ & & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & \boxed{0} \end{array}$$

$x^2 + x + 1$

$$\frac{x^3-1}{x-1} = \boxed{x^2 + x + 1}$$

4) $\dfrac{2x^2 - 7x + 8}{2x - 3}$ ← coefficient needs to be 1 $\Rightarrow x - \frac{3}{2}$

$$\begin{array}{c|ccc} \frac{3}{2} & 1 & -\frac{7}{2} & 4 \\ & & \frac{3}{2} & -3 \\ \hline & 1 & -2 & \boxed{1} \end{array}$$

← Divide all coefficients by 2!

remainder → $\dfrac{1}{x-\frac{3}{2}} \cdot \dfrac{2}{2} = \dfrac{2}{2x-3}$

$x - 2 + \dfrac{2}{2x-3}$

$$\frac{2x^2-7x+8}{2x-3} = \boxed{x - 2 + \frac{2}{2x-3}}$$

FACTORIZATION OF $X^N - A^N$

4) Use synthetic division to divide $x^5 - a^5$ by $x - a$.

$$\begin{array}{c|cccccc} a & 1 & 0 & 0 & 0 & 0 & -a^5 \\ & & a & a^2 & a^3 & a^4 & a^5 \\ \hline & 1 & a & a^2 & a^3 & a^4 & \boxed{0} \end{array}$$

$x^4 + ax^3 + a^2x^2 + a^3x + a^4$

$$x^5 - a^5 = (x-a)(x^4 + ax^3 + a^2x^2 + a^3x + a^4)$$

6) Factor: $x^4 - 256$

$x^4 - 256 = x^4 - 4^4$

$x^4 + 0x^3 + 0x^2 + 0x - 256$

$$\begin{array}{c|ccccc} 4 & 1 & 0 & 0 & 0 & -256 \\ & & 4 & 16 & 64 & 256 \\ \hline & 1 & 4 & 16 & 64 & \boxed{0} \end{array}$$

$x^3 + 4x^2 + 16x + 64$

$$x^4 - 256 = (x-4)(x^3 + 4x^2 + 16x + 64)$$